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# CONSTRUCTION OF EFFECTIVE COMPUTING SCHEMES IN MAPLE FOR SOLUTION OF THE PROBLEM OF DETERMINATION OF ULTIMATE STRAIN DURING DIFFICULT STRAIN

*There had been considered the problem of determination of ultimate strain of difficult strain which occurs on a free lateral surface at a axial-symmetric compression with variation of friction conditions on end faces. There had been highlighted the key moments of this problem. It had been shown that application of standard commands of system Maple for the sokution of the considered problem is not efficient from the point of view of time spent for carrying out calculations. There had been analysed different approaches to construction of efficient computing schemes. It had been shown that the most efficient is the schema which is based on application of the combined method of chords and tangents.*

***Keywords:*** *tensely and stained state; slip of cylindrical pattern; accumulated strain, destruction model; iterative methods.*

### Task setting and analysis of the latest researches

Object of research. Processes of difficult strain.

Subject of research. Mathematical model of accumulated strain on a free lateral surface with axial slip of cylindrical pattern with different conditions a friction on end faces.

Research objective. Search of effective computing schemas in the environment of mathematical package Maple for task of determination of the ultimate strain of difficult strains which arises during the axial slip of cylindrical billets with different conditions of a friction on end faces.

Research problems. 1. A choice of model of accumulation of damages. 2. Determination of model of *tensely and stained state* on a free lateral surface during slip of cylindrical pattern with different conditions of friction on end faces. 3. Setting mathematical task of determination of ultimate strain of difficult strain and research of alternative methods for its solution in the environment of Maple. 4. Determination of efficient schema of the solution of the task and evident representation of dependence of ultimate strain on the process of change of *tensely and stained state* of critical area of billet.

Urgency of the work. Solution of tasks of the theory of deformability leads to great volume of calculations [1-3]. And despite of capacity of modern computing means, in some cases the problem of elaboration of effective computing schemas remains actual.

### Major part

According to classification which is offered in [1], a difficult strain we will name such a process of no monotonic strain when the components of the directing tensors of increase of plastic strains change continuously. A number of models had been developed for the determination of ultimate strain of difficult strain within the theory of continuous damages [1, 2, 4, 5]. We will consider the simplest variant: model which is based on a linear principle of accumulation of damages, - Kholmogorov's model [4, 2, 1]:

$$\psi(\varepsilon_u) = \int_0^{\varepsilon_u} \frac{d\varepsilon_u}{\varepsilon_{*c} \cdot [\eta(\varepsilon_u)]}, \tag{1}$$

where $\psi$ - damage of macro particle which changes from 0 in an initial condition to 1 when achieved the limiting state; $\varepsilon_u$ - accumulated plastic strain:

$$\varepsilon_u(t) = \int_0^t \dot{\varepsilon}_u(\tau) d\tau, \tag{2}$$

where $\tau$, $t$ – time; $\dot{\varepsilon}_u$ – deformation rate intensity; $\varepsilon_{*c} = \varepsilon_{*c}(\eta)$ – curve of ultimate strains for condition of stationary strains; $\eta$ – index of strained condition [1, 2, 6].

Taking into account (1) the condition of achievement of limiting state takes a form:

$$\psi(\varepsilon_*) = 1, \tag{3}$$

or

$$\int_0^{\varepsilon_*} \frac{d\varepsilon_u}{\varepsilon_{*c} \cdot [\eta(\varepsilon_u)]} = 1, \tag{4}$$

where $\varepsilon_*$ – ultimate strain for condition no monotonic, in particular, difficult strain.

As is seen from the representation (4) finding of ultimate strain $\varepsilon_*$ is reduced to the solution of the nonlinear equation. Nonlinear function is presented by certain integral with a variable top limit. It is necessary to notice, that internal simple equation (4) may in some cases lead to complex correlations and in turn to considerable time for carrying out of calculations for decision making.

For determination of the *tensely and stained state* on a free lateral surface on axial slip of cylindrical patterns we will take advantage of results which are resulted in works [7, 8]. A curve of ultimate strain for stationary strain we will approximate by expression:

$$\varepsilon_{*c}(\eta) = \varepsilon_{*c}(\eta = 0) \cdot \left( \frac{(1-\eta) \cdot \varepsilon_{*c}(\eta = -1)}{2 \cdot \varepsilon_{*c}(\eta = 0)} + \frac{(1+\eta) \cdot \varepsilon_{*c}(\eta = 0)}{2 \cdot \varepsilon_{*c}(\eta = 1)} \right)^{-\eta}, \tag{5}$$

where $\varepsilon_{*c}(\eta = -1), \varepsilon_{*c}(\eta = 0), \varepsilon_{*c}(\eta = 1)$ – ultimate strain of compression, torsion, tension correspondingly.

Taking into account numerical values parameters of model, the equation (4) can be presented in as follows $\left( \alpha \in \left[ 0, \frac{\pi}{2} \right[, m > 0 \right)$

$$m \cdot \int_0^{\alpha} \frac{1{,}47 \cdot \sqrt{3 + \dfrac{1}{\cos^4 x}}}{\mathring{a}^{\left( -\frac{(1-3\cdot\cos^2 x)\cdot \ln\left( \frac{0{,}94\cdot(1-3\cdot\cos^2 x)}{\sqrt{(1+3\cdot\cos^4 x)}} \right)}{\sqrt{(1+3\cdot\cos^4 x)}} \right)}} dx - 1 = 0, \tag{6}$$

where *m* – reflects friction conditions on end faces and it is considered as constants in the course of research of the specific pattern [6, 8]; *α* - parameter of slipping process.

Root of the equation (6) we will designate through $\alpha_*$, $\alpha_* \in \left] 0, \frac{\pi}{2} \right[$.

Let's note that correlations (5), (6) are presented for the first time in the given work. The other works will be dedicated to the methodic for receiving these equations.

The task of determination of ultimate strain under conditions axial slip of cylindrical patterns with different conditions of friction on end faces is reduced to the decision of the equation (6) for specific values *m*. After finding corresponding values $\alpha_*$ the co-ordinates of a limiting point on a corresponding way of stain $\eta(\varepsilon_u) = \eta[\varepsilon_u(\alpha)]$ can be easily found. This work will consider such alternative ways of the decision of the assigned tasks: 1. Direct application of a standard command of system Maple for receiving numerical decision of the nonlinear equation; 2. Decomposition of integrand function in power series with the subsequent integration which should raise speed of calculation of values of the researched nonlinear function; 3. Application of iterative methods of a numerical finding of the decision of the nonlinear equation.

System Maple provides the command fsolve for the numerical solution of one equation or system of the equations (linear or nonlinear).

For calculation of values of the left part of the equation (6) with specific values *m*, *α* we will create procedure PsK (*m, α*):

```
> PsK:=
(m,alpha)->evalf(m*Int(1.47*sqrt(3+1/(cos(x)^4)/exp(...),x=0..
alpha)-1);
```

For the economy of place and prevention dispersion of attention, the obvious part of program code is omitted. This procedure realises numerical finding of certain integral by means of a standard command evalf. The principal part of the procedure is the fact that the command of finding integral is written down from the big letter in the inert form. In such a case the system will apply the numerical algorithm of finding integral. In the opposite case, when the integral is set by command int, the system will try to find at first an integral in a symbolical kind, which will demand certain time, and only after that will make sure, that it can not find the antiderivative, and will automatically connect algorithm of numerical integration.

For the application of the command fsolve also we will create procedure `Розв_к`:

```
> `Розв_к`:=proc(m,`діапазон`:: range)
if type(m,numeric) then
  fsolve(PsK(m,t)-1,t, `діапазон`)
else
  'procname'(m,`діапазон`)
end if
end proc;
```

It is necessary to notice, that availability in procedure ` Розв_к ` design with the operator ' procname ' (m, ` діапазон `) for situation processing in a case not numerical value of parameter *m,* is key one for building curve of function $\alpha_* = \alpha_*(m)$.

For the specific value *m* the system carries out calculation of root, approximately, for 7 sec., which is easy to determine by means of a design:

```
> `Старт`:= time():`Розв_к`(0.09,0..1.5); `Фініш`:=time() -
`Старт`;
```

$$1.380775711$$

$$T\_k \quad 7.344$$

For building the graph $\alpha_* = \alpha_*(m)$ in the range $m \in [0{,}01{,}8]$ the system spends more than 20 minutes (!!!). In modern conditions such a situation is unacceptable, as, in particular, time for solution is not correlated with degree of complexity of a problem. All calculations were made on the computer with the central processor: AMD Sempron, 1500 MHz (9 x 167) 2200 +.

The approach which is based on decomposition of an integrand function in power series with further integration, looks nice in connection with ease of realization of these operations in the environment of system Maple. Was supposed, that replacement with a polynomial of complex function which is determined by certain integral with a variable top limit, should accelerate a numerical finding of root of the nonlinear equation

By means of commands taylor (*f (x), x=0,17); convert (%, polynom), x*) where *f (x)* – integrand function in the ratio (6), we will have a corresponding polynomial:

$$0{,}77 + 1{,}11 \cdot x^2 + 1{,}38 \cdot x^4 + 1{,}32 \cdot x^6 + 1.0 \cdot x^8 + 0.59 \cdot x^{10} + 0.23 \cdot x^{12} + 0{,}01 \cdot x^{14} - 0{,}07 \cdot x^{16} \quad (7)$$

Fig. 1 presents comparison of curves of integrand function *f (x)* and its decomposition in series (7) to point vicinities *x=0*:

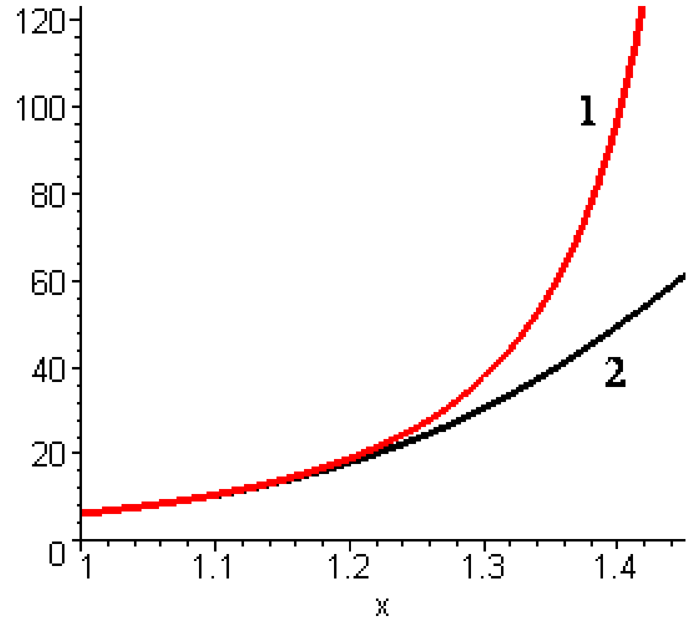


Fig. 1. The comparison of curves of integrand function f (x) and its decomposition in series (7) to point vicinities x=0,where 1 – function f (x); 2 – polynom (7)

As is seen, when *x*>1,18 it is impossible to find the coordination as satisfactory. The increasing in degree of a polynomial and changing point in the vicinity of which the decomposition takes place, did not allow to achieve the substantial improvement of the results. Using approximation (7) equation roots (6) with satisfactory accuracy come true for *m*> 0,18. In this case the attention os paid not to the speed of calculation, but to the unsatisfactory accuracy of obtaining results.

Iterative methods are de-facto most widespread methods for receiving of solutions of the nonlinear equations. Methods of chords, tangents and secants are the leading among iterative methods.

Basic difference of a method of secants [9] from a method of tangents is that in the first method is continually calculated only new value of function, when in the second method on each step the value of function together with its derivative are calculated. Therefore, despite of less fast convergence of a method of secants in comparison with a method of tangents, due to less number of calculations on each step, the first method can appear to be more efficient. Whereas in the considered case the function represents certain integral with a variable top limit, the time of calculation of derivative values of function is not enough in comparison with calculation of values of the function. The above allows to reject a method of secants and to concentrate on methods of chords and tangents.

Each of these methods suffers the same drawback, namely, an approach to a root occurs on the one hand. This fact complicates construction of criterion of finishing iterations.

Application of the combined method of chords and tangents allows receiving a convenient sign of achievement of the set accuracy: each of these methods generates sequence of values which come nearer to a root from the different parties. Calculations finish on condition that the difference between current approaches to a root, which are calculated on two methods, becomes less than demanded degree of accuracy.

For application of a method of chords it is necessary to find a segment , which contains a root and on which function is monotonous. For application of a method of tangents it is necessary to allocate a piece which contains a root and on which function is not only monotonous, but also has no points of inflection. It is possible to prove, that function, which is determined by the left part of the equation (6) is monotonously increasing and bent on a range of definition with any values *m*> 0. Authors found the proof of this fact, which will be described in separate work. Consequently, following the method of chords the approach to a root will occur from left, and following the method of tangents – from right.

Realization in the environment of system Maple of a method of tangents (Newton's method) [10] is significant. We will describe a procedure which, on the set function, automatically generates the iteration formula.

```
> `М_ньютон`:=proc(y::procedure)
  ((m,x)->x)-eval(y)/D[2](eval(y))
end proc:
```

In this case it is easy to write down the iteration formula:

$$\alpha_{n+1} = \alpha_n - \frac{F(m,\alpha_n)}{f(\alpha_n)}, \tag{8}$$

where $\alpha_n$ - current approach of a root $\alpha_*$ ratio (6); $F$ ($m$, $\alpha_n$) - the left part of equality (6); $f(x)$ - integrand function in the ratio (6).

In the majority of other cases the above procedure releases from the necessity of performance of differentiation operations, which sometimes is a complex one.

The command

```
> `Колм_Ньют`:=`М_Ньютон`(PsK);
```

allows to receive function for calculation by a method of tangents of the current approached value of a root of the equation (6).

It is necessary to notice, that additional “complication” can lead to appreciable deterioration of efficiency of calculations. We realize a method of chords by following procedures:

```
> `М_хорд`:=proc(F::procedure,m,x1,x2)
    evalf(x1*F(m,x2)-x2*F(m,x1))/(F(m,x2)-F(m,x1));
  end proc:
```

We receive specific expression for function PsK

```
> `Колм_хорд`:=`hord`(PsK,m,X1,X2);
```

We transform expression into function

```
> `Колм_хорд`:=unapply(`Колм_хорд`,m,x1,x2);
```

Expressions, which are displayed as a result of work of these procedures well not be described since this will lead to additional complexity of expressions. Having reproduced the above commands in the environment of Maple, these expressions can be seen. Expression, which is used in procedure for calculation current approximation of a root by a method of chords, contains some elements which repeat.

In particular, expression

$$\sqrt{1+3\cdot\cos^4 x}$$

Is met three times and each time is calculated anew. From the point of view of programming elements such situation is beneath criticism. But this is another proof that when it is a matter of numerical calculations, it is necessary to adhere to these elements, despite of rapid growth of computing power of modern computers. In this case for achievement of accuracy 0,005 when $m=2$ *by* the combined method it required to execute 80 (!) steps, and already for achievement of accuracy 0,0005 it required 788 steps. And the analysis of results showed that such big quantity of iterations for the combined method is connected with current approximations, which are calculated by a method of chords. The method of tangents gave result already on the fourth iteration, which exceeded the set accuracy and remained invariable on all following iterations. The authors did not explain the reason for it since the situation in this case had been created artificially.

For construction the graphics of curve of ultimate strain during slipping cylindrical patterns with different conditions of friction on end faces we will create the following procedure

```
> `Графік`:=proc(m,X1,X2,dx)
local a,b,x0;
```

```
if type(m,numeric) then
  a:=X1:b:=X2:x0:=X2:i:=0:
  while (b-a)>=dx do
    x0:=`Колм_Ньют`(m,x0);
    b:=x0;
    a:=`hord`(PsK,m,a,b);
  end do:
  (a+b)/2
else
  'procname'(m,X1,X2,dx)
end if
end proc:
```

The above procedure is key one for possibility of construction of graphic presented on fig. 2:

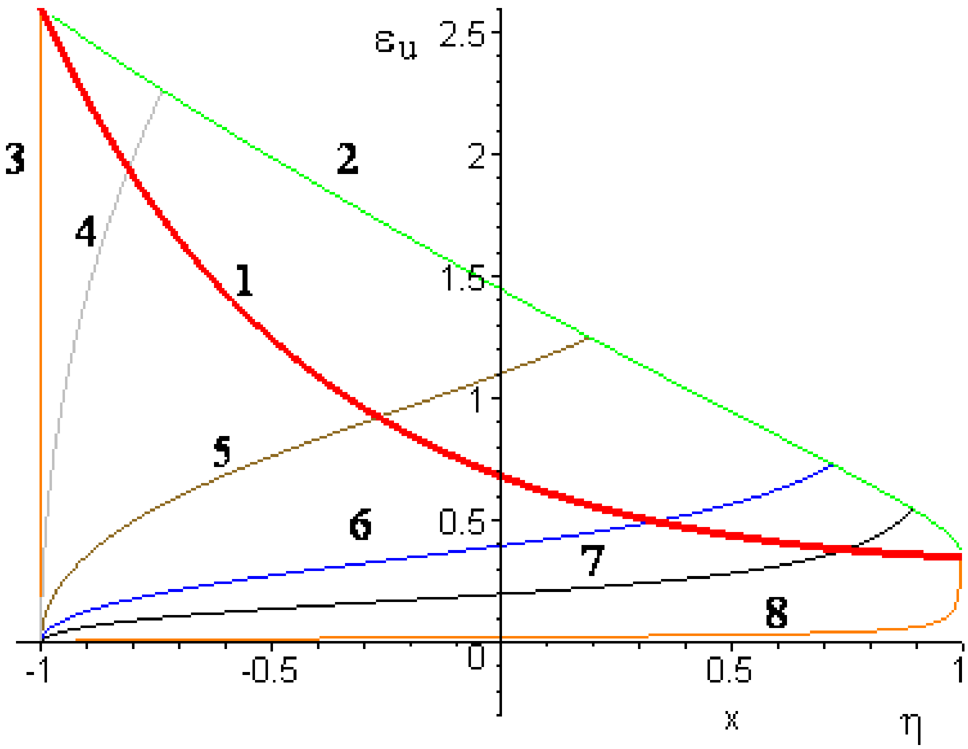


Fig. 2. Modeling of ultimate strains during axial slipping cylindrical patterns: 1 – curve of ultimate strain for stationary strain; 2 – curve of ultimate strains for difficult strain; 3÷8 - curves which display ways of strain with different conditions of a friction at end faces: 3 – m = 30, 4 – m = 2, 5 – m = 0,5, 6 – m = 0,18, 7 – m = 0,09, 8 – m = 0,01

Duration for construction of the given graphic by system does not exceed 20 sec., that is completely comprehensible. At the same time, necessity of application instead of representation (1) of the models, which are based on the nonlinear law of summation of the damages, in particular offered in [2], will lead to essential increase in volume of calculations. It is connected with the fact that under the integral in the ratio (6) there is a value which is represented as the integral. Application of tensor models [1] will lead still to greater number of calculations. As a result there may appear the necessity in perfection of the schema which is built in the given work. We will note that the curve of ultimate strain for conditions of difficult strain is constructed for the first time in the given work. Such curves have previously been built by means of interpolation and extrapolation on 3÷5 points [2, 8].

## Conclusions

1. There had been built schema of calculations, based on application of the combined method of

chords and tangents. In comparison with the application of standard a standard command of system Maple, it will allow to raise speed of building of graphic presented on fig. 2, in 60 times, having reduced duration of calculations from 20 minutes to 20 sec.

2. The developed technique is suitable not only for the considered process of slipping cylindrical patterns, but also for any process which is accompanied by difficult strain.

3. Attraction of nonlinear and tensor models of accumulation of damages may cause perfection of the constructed schema of calculations.

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