B. I. Mokin, Dr. Sc. (Eng); V. V. Kaminsky, Cand. Sc. (Eng)

METRIC IN SPACE OF THE DIRECTED LEVELS OF MEMBERSHIPS OF POORLY SET PARAMETERS OF COMPLEX SYSTEMS

The work introduces metric of special kind in the space of directed levels of membership of weak set. The suggested metric allows simplifying the process of simulation and analysis of undetermined parameters of complex system using poor sets.

Keywords: *weak set, belonging, direction, metric, directed level, directed axis.*

In works [1-6] authors suggested the new approach to the simulation of complex systems on the basis of the apparatuses of the theory of weak sets developed by them, which allows to simulate indefinite parameters of systems under conditions of absence of both, the numerical, as well as fuzzy or linguistic values of indefinite parameters. The basic tool of the new technology of complex system simulation in conditions of data uncertainty is the X weak set $\tilde{\tilde{A}}$ set in a universum [2, 3, 6] which unlike indistinct set $\tilde{A}$ is set not by membership function μ_A: $X \to M_\alpha$, but by function of levels ν_A: $X \to M_{\alpha\omega}$, where $M_{\alpha\omega} = M_\alpha \times \{+; -\} \setminus \{(\vee M_\alpha; -)\}$ - space of the directed levels of membership which is the Cartesian product of space of not directed (usual) levels of membership M_α and spaces of direction $\{+; -\}$, $\vee M_\alpha$ - a maximum element of space M_α. Elements of this space which are the ordered pares of the type $(\alpha; +)$, $(\beta; -)$, $\alpha, \beta \in M_\alpha$ are called positively (for a case $(\alpha; +)$) and negatively (for a case $(\beta; -)$) directed levels of a membership, it is convenient to designate accordingly α^+, β^-. Function of levels of weak set does not set to the elements of the universum any specific degrees membership to weak set, but only the directed levels of a membership. Thus positively directed levels of a membership according to [4] can be interpreted, as the lower exact edge of possible values of degrees of a membership of corresponding elements of a universum. Thus the least upper bound of a membership of these elements in any way is not regulated and limited only to a universal element $\vee M_\alpha$ spaces of not directed levels of a membership. Negatively directed levels of a membership in turn can be interpreted, as the upper exact edge of possible values of degrees of a membership. Thus the lower exact edge of memberships of elements of a universum in any way is not set and limited only to the minimum element $\wedge M_\alpha$ spaces M_α.

In space of not directed levels of a membership the usual weak linear order, and in space of directednesses - a linear order $\{(+, +), (+, -), (-, -)\}$ takes place. As for the space $M_{\alpha\omega}$, there had been set the special strict linear order $S_{\alpha\omega}$ so that

$$S_{\alpha\omega} = \forall\, (\alpha, \omega_\alpha), (\beta, \omega_\beta) \in M_{\alpha\omega}\, ((\beta, \omega_\beta) > (\alpha, \omega_\alpha) \Leftrightarrow (\beta > \alpha \wedge \omega_\beta = \omega_\alpha) \vee (\omega_\beta > \omega_\alpha)), \quad (1)$$

And the diagonal ration on $M_{\alpha\omega}$ sets equality of the directed levels of a membership

$$\forall\, (\alpha, \omega_\alpha), (\beta, \omega_\beta) \in M_{\alpha\omega}\, ((\alpha, \omega_\alpha) = (\beta, \omega_\beta) \Leftrightarrow \alpha = \beta \wedge \omega_\alpha = \omega_\beta). \quad (2)$$

The work [5] presents the geometrical interpretation of space of the directed levels of a membership with the linear order set on it (1). According to this interpretation, space $M_{\alpha\omega}$ can be represented in the form of a segment of the straight line, points of which set the directed levels of membership. Such segment is named as an axis of the directed levels of a membership. Fig. 1 presents the axis of the directed levels of a membership for case $M_\alpha = [0; 1]$. The direction of growth of values of the directed levels of a membership according to strict linear order $S_{\alpha\omega}$ is shown in drawing by dashed arrows.

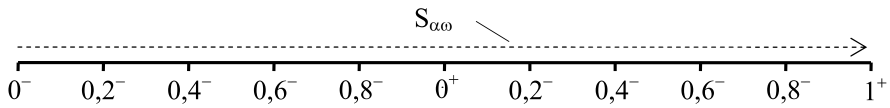

Fig. 1. An axis of the directed levels of a membership at $M_{\alpha} = [0; 1]$

As is seen in fig. 1, a linear order in space $M_{\alpha\omega}$ with negatively and positively directed levels of a membership essentially differs from a usual linear order on a segment of a numerical axis with negative and positive numbers. Therefore the usual Euclidean metric of a numerical straight line is not suitable for distance definition between arbitrary pair points of the directed axis of levels of a membership.

It is known, that many fundamental notions and results of mathematical analysis are connected not with the algebraic nature of real numbers, but only with concept of distance of space of real numbers, i.e. with set of real numbers, as metric space. The basic notions and results of the theory of limits, notion of a continuity and smoothness of functions etc belong to such notions and facts. To introduce similar concepts in the theory of weak sets and, in particular, important to be used in this theory notions of continuous and explosive, decreasing and increasing functions of levels of weak set, it is necessary to set the metric in space of the directed levels of a membership.

In order to emphasize which metric is used, we will designate metric space in the form of the pair $\langle M_{\alpha\omega}, \rho_M \rangle$, where ρ_M - the metric on $M_{\alpha\omega}$, i.e. a nonnegative real-valued function which for each pair of elements from $M_{\alpha\omega}$ sets distance between them. With the known metric ρ_M we will name also set $M_{\alpha\omega}$ as the metric space, on the elements of which the metric is set as it is usually accepted [7].

It is obvious that metric in space $M_{\alpha\omega}$ can be introduced by different ways. We will introduce the metric ρ_M on an axis of the directed levels of a membership so that for any pair of points of this axis between them it is possible to measure distance by means of a usual ruler, and in case of unidirectional levels of a membership it coincides with the usual metric ρ_R of real numbers

$$\rho_R: R^2 \to R_{+0},\ \rho_R(a, b) = |a - b|,\ \forall a, b \in R, \text{ i.e.}$$

$$\forall \alpha, \beta \in M_{\alpha} (\rho_M(\alpha^+, \beta^+) = \rho_M(\alpha^-, \beta^-) = \rho_R(\alpha, \beta)), \tag{3}$$

Where $R_{+0} = \{0\} \cup R_+$; R_+ - set of positive real numbers.

With this in view we will consider function ρ_M: $M^2_{\alpha\omega} \to R_{+0}$ in space $M_{\alpha\omega}$ such, that

$$\forall \alpha, \beta \in M_{\alpha}\ \forall \omega, \psi \in M_{\omega} (\rho_M(\alpha^{\omega}; \beta^{\psi}) = |\alpha - \beta| \Leftrightarrow \omega = \psi); \tag{4}$$

$$\forall \alpha, \beta \in M_{\alpha} (\rho_M(\alpha^+; \beta^-) = |\alpha - \wedge M_{\alpha}| + |\beta - \vee M_{\alpha}|); \tag{5}$$

$$\forall \alpha, \beta \in M_{\alpha} (\rho_M(\alpha^-; \beta^+) = |\alpha - \vee M_{\alpha}| + |\beta - \wedge M_{\alpha}|). \tag{6}$$

Let's show that function ρ_M is the metric in space $M_{\alpha\omega}$ for which the condition (3) is satisfied.

According to definition of metric space [7] function ρ_M will be the metric only when it is a nonnegative real-valued function satisfying to all axioms of metric spaces. In our case that can be noted in the form:

$$\forall \alpha^{\omega}, \beta^{\psi} \in M_{\alpha\omega} (\rho_M(\alpha^{\omega}; \beta^{\psi}) = 0 \Leftrightarrow \alpha^{\omega} = \beta^{\psi}); \tag{7}$$

$$\forall \alpha^{\omega}, \beta^{\psi} \in M_{\alpha\omega} (\rho_M(\alpha^{\omega}, \beta^{\psi}) = \rho_M(\beta^{\psi}, \alpha^{\omega})); \tag{8}$$

$$\forall \alpha^{\omega}, \beta^{\psi}, \gamma^{\chi} \in M_{\alpha\omega} (\rho_M(\alpha^{\omega}, \gamma^{\chi}) \le \rho_M(\alpha^{\omega}, \beta^{\psi}) + \rho_M(\beta^{\psi}, \gamma^{\chi})). \tag{9}$$

It may be noticed, that ρ_M is a nonnegative real-valued function as its values according to (4) - (6) are an absolute value of a real number or the sum of absolute values of real numbers. Besides, according to (4) on set of unidirectional levels of a membership of value of function ρ_M for any pair of such levels do not depend in any way on their directedness. Taking it into account, and also considering condition (4), it actually sets the metric in space of not directed levels of membership

M_α which coincides with the usual metric of real numbers ρ_R.

Since usual metric ρ_R is the rigorous metric on set of real numbers, than on the set of not directed levels of a membership which is a subset of set of real numbers, function ρ_M is also the metric one, which in case of unidirectional levels of a membership corresponds to the usual metric of real numbers ρ_R. From here we conclude, that the equality (4) answers all axioms of the metric (7) - (9).

Let's show further that equalities (5), (6) also answer axioms of the metric (7) - (9). At first we will show, that these equalities satisfy an axiom (7).

Let us assume that the axiom (7) for these equalities is not fulfilled. In that case there should be such pair of the directed levels of a membership α^ω, β^ψ, that

$$\alpha^\omega = \beta^\psi \ \wedge\ \rho_M(\alpha^\omega, \beta^\psi) \neq 0, \text{ or} \tag{10}$$

$$\alpha^\omega \neq \beta^\psi \ \wedge\ \rho_M(\alpha^\omega, \beta^\psi) = 0. \tag{11}$$

Since the conditions (5), (6) concern only the case, when the directions of levels of a membership are opposite, than the expressions (10), (11) for this case is possible to rewrite in such an expanded form:

$$\alpha^\omega = \beta^\psi \ \wedge\ \omega \neq \psi \ \wedge\ \rho_M(\alpha^\omega, \beta^\psi) \neq 0, \text{ or} \tag{12}$$

$$\alpha^\omega \neq \beta^\psi \ \wedge\ \omega \neq \psi \ \wedge\ \rho_M(\alpha^\omega, \beta^\psi) = 0. \tag{13}$$

But the case (12) is impossible because according to the ration of equality (2) in space $M_{\alpha\omega}$ the directed levels of a membership with the equal not directed memberships and opposite directions cannot be equal.

Let's consider the case (13). According to (5), (6) the following biconditionals should be identically true:

$$\rho_M(\alpha^+; \beta^-) = 0 \Leftrightarrow |\alpha - \wedge M_\alpha| + |\beta - \vee M_\alpha| = 0; \tag{14}$$

$$\rho_M(\alpha^-; \beta^+) = 0 \Leftrightarrow |\alpha - \vee M_\alpha| + |\beta - \wedge M_\alpha| = 0. \tag{15}$$

But the sum of absolute values of two any real numbers equals zero only when both absolute values equal zero. Therefore the statement (14), (15) is possible to write in such equivalent form:

$$\rho_M(\alpha^+; \beta^-) = 0 \Leftrightarrow |\alpha - \wedge M_\alpha| = 0 \wedge |\beta - \vee M_\alpha| = 0, \tag{16}$$

$$\rho_M(\alpha^-; \beta^+) = 0 \Leftrightarrow |\alpha - \vee M_\alpha| = 0 \wedge |\beta - \wedge M_\alpha| = 0. \tag{17}$$

Right members of statements (16), (17) are possible to rewrite according to such chains equivalences:

$|\alpha - \wedge M_\alpha| = 0 \wedge |\beta - \vee M_\alpha| = 0 \equiv \alpha - \wedge M_\alpha = 0 \wedge \beta - \vee M_\alpha = 0 \equiv \alpha = \wedge M_\alpha \wedge \beta = \vee M_\alpha$, and accordingly $|\alpha - \vee M_\alpha| = 0 \wedge |\beta - \wedge M_\alpha| = 0 \equiv \alpha - \vee M_\alpha = 0 \wedge \beta - \wedge M_\alpha = 0 \equiv \alpha = \vee M_\alpha \wedge \beta = \wedge M_\alpha$.

Substituting instead of a right member biconditional (16), (17) the received conjunction, we will have: $\rho_M(\alpha^+; \beta^-) = 0 \Leftrightarrow \alpha = \wedge M_\alpha \wedge \beta = \vee M_\alpha$, $\rho_M(\alpha^-; \beta^+) = 0 \Leftrightarrow \alpha = \vee M_\alpha \wedge \beta = \wedge M_\alpha$.

But for the directed level β^- the equality $\beta = \vee M_\alpha$ can take place only in case, when $\beta^- = (\vee M_\alpha; -)$, and for the directed level α^- the equality $\alpha = \vee M_\alpha$ can take place only in case, when $\alpha^- = (\vee M_\alpha; -)$. At the same time according to the definition of space of the directed levels of a membership the directed level $(\vee M_\alpha;-)$ in it spacious is absent. Thus having assumed, that the axiom (7) is not fulfilled, we have come to false conclusions (14), (15). It means, that the made supposition is false, and equalities (5), (6) answer an axiom (7).

As for the axiom (8), the correspondence of equalities (5), (6) to these axiom directly follows from a commutability of the sum of absolute values of real numbers. Really, from (5), (6) it follows, that

$$\rho_M(\alpha^+; \beta^-) = |\alpha - \wedge M_\alpha| + |\beta - \vee M_\alpha|,$$
$$\rho_M(\beta^-; \alpha^+) = |\beta - \vee M_\alpha| + |\alpha - \wedge M_\alpha|,$$

$$\rho_M\ (\alpha^-;\ \beta^+) = |\ \alpha - \vee M_\alpha\ | + |\ \beta - \wedge M_\alpha\ |,$$
$$\rho_M\ (\beta^+;\ \alpha^-) = |\ \beta - \wedge M_\alpha\ | + |\ \alpha - \vee M_\alpha\ |.$$

But, as

$$|\ \alpha - \wedge M_\alpha\ | + |\ \beta - \vee M_\alpha\ | = |\ \beta - \vee M_\alpha\ | + |\ \alpha - \wedge M_\alpha\ | \text{ i}$$
$$|\ \alpha - \vee M_\alpha\ | + |\ \beta - \wedge M_\alpha\ | = |\ \alpha - \vee M_\alpha\ | + |\ \beta - \wedge M_\alpha\ |,$$

then $\rho_M\ (\alpha^+;\ \beta^-) = \rho_M\ (\beta^-;\ \alpha^+)$ i $\rho_M\ (\alpha^-;\ \beta^+) = \rho_M\ (\alpha^-;\ \beta^+)$. i.e. the axiom (8) is fulfilled for equalities (5), (6).

It is necessary to show, that equalities (5), (6) also satisfy the axiom (9). First, we will fulfill the proof for the equality (5). As the equality (5) sets distance between two differently directed levels of membership, the initial level has a positive direction, and the final – negative, it is necessary and sufficient to fulfill the proof for two cases of the possible ordered gangs of three directed levels of a membership which differ by directions of an average level of a membership, and first and last levels keep the directions. Thus gangs of three directed levels of a membership which should be considered, will look like: $\alpha^+,\ \beta^+,\ \gamma^-$ and $\alpha^+,\ \beta^-,\ \gamma^-$.

First, let's consider the first gang of the directed levels of a membership. According to (4) $\rho_M\ (\alpha^+;\ \beta^+) = |\ \alpha - \beta\ |$, and according to (5)

$$\rho_M\ (\alpha^+;\ \gamma^-) = |\ \alpha - \wedge M_\alpha\ | + |\ \gamma - \vee M_\alpha\ |; \tag{18}$$

$$\rho_M\ (\beta^+;\ \gamma^-) = |\ \beta - \wedge M_\alpha\ | + |\ \gamma - \vee M_\alpha\ |.$$

Then

$$\rho_M\ (\alpha^+;\ \beta^+) + \rho_M\ (\beta^+;\ \gamma^-) = |\ \alpha - \beta\ | + |\ \beta - \wedge M_\alpha\ | + |\ \gamma - \vee M_\alpha\ |. \tag{19}$$

From (18) and (19) it follows that an inequality

$$\rho_M\ (\alpha^+;\ \gamma^-) \le \rho_M\ (\alpha^+;\ \beta^+) + \rho_M\ (\beta^+;\ \gamma^-)$$

It is fulfilled only when

$$|\ \alpha - \wedge M_\alpha\ | + |\ \gamma - \vee M_\alpha\ | \le |\ \alpha - \beta\ | + |\ \beta - \wedge M_\alpha\ | + |\ \gamma - \vee M_\alpha\ | \equiv$$
$$\equiv |\ \alpha - \wedge M_\alpha\ | \le |\ \alpha - \beta\ | + |\ \beta - \wedge M_\alpha\ |.$$

Let us convince, that last inequality is really fulfilled. From properties of an absolute value of a real number it follows that

$$|\ \alpha - \beta\ | + |\ \beta - \wedge M_\alpha\ | \ge |\ \alpha - \beta + \beta - \wedge M_\alpha\ | \equiv |\ \alpha - \beta\ | + |\ \beta - \wedge M_\alpha\ | \ge |\ \alpha - \wedge M_\alpha\ |.$$

Thus for the first set of the directed levels of a membership the equality (5) satisfies the axiom (9).

For the second set of levels of a membership $\alpha^+,\ \beta^-,\ \gamma^-$ according to (5) we will receive

$$\rho_M\ (\alpha^+;\ \gamma^-) = |\ \alpha - \wedge M_\alpha\ | + |\ \gamma - \vee M_\alpha\ |, \tag{20}$$

$$\rho_M\ (\alpha^+;\ \beta^-) = |\ \alpha - \wedge M_\alpha\ | + |\ \beta - \vee M_\alpha\ |.$$

For the same case according to (4) we will have

$$\rho_M\ (\beta^-;\ \gamma^-) = |\ \beta - \gamma\ | = |\ \gamma - \beta\ |.$$

From the last two equalities we will receive:

$$\rho_M\ (\alpha^+;\ \beta^-) + \rho_M\ (\beta^-;\ \gamma^-) = |\ \alpha - \wedge M_\alpha\ | + |\ \beta - \vee M_\alpha\ | + |\ \gamma - \beta\ |. \tag{21}$$

From (20), (21) it follows that an inequality $\rho_M\ (\alpha^+;\ \gamma^-) \le \rho_M\ (\alpha^+;\ \beta^-) + \rho_M\ (\beta^-;\ \gamma^-)$ is fulfilled only if the inequality is fulfilled

$$|\alpha - \wedge M_\alpha| + |\gamma - \vee M_\alpha| \le |\alpha - \wedge M_\alpha| + |\beta - \vee M_\alpha| + |\gamma - \beta| \equiv |\gamma - \vee M_\alpha| \le |\gamma - \beta| + |\beta - \vee M_\alpha|.$$

But the last inequality is really fulfilled. It is easy to be convinced of it, considering, that

$$|\ \gamma - \beta\ | + |\ \beta - \vee M_\alpha\ | \ge |\ \gamma - \beta + \beta - \vee M_\alpha\ | \equiv |\ \gamma - \beta\ | + |\ \beta - \vee M_\alpha\ | \ge |\ \gamma - \vee M_\alpha\ |.$$

Thus for all possible gangs of the directed levels of a membership the equality (5) satisfies the axiom (9).

For proof completion we will show, that for equality (6) the same conclusion takes place - it also satisfies the axiom (9). As the equality (5) satisfies all the axioms of metric spaces it also satisfies the separate axiom of the metric (8). Taking into account this axiom and a commutability of the sum of absolute values of real numbers we will rewrite it in such equivalent form

$$\rho_M\,(\beta^{-}; \alpha^{+}) = |\,\beta - \vee M_{\alpha}| + |\,\alpha - \wedge M_{\alpha}|.$$

But the received equality with the accuracy to mutual substitution of numerals of the directed levels of a membership is equivalent to equality (6). From here we conclude, that the equality (6), as well as equality (5) satisfies the axiom (9).

Thus it is proved, that all equalities (4) - (6) satisfy the axioms of the metric (7) - (9), and it means, that function ρ_M is the metric which sets distance between arbitrary pair points of space $M_{\alpha\omega}$. And the distance between any levels of a membership with an identical directions set by the metric ρ_M is equal to distance between the corresponding not directed levels of a membership set by the usual Euclidean metric ρ_R, that is guaranteed by a condition (3). However for any pair of levels of a membership with different directions the metric ρ_M sets distance according to the specific laws (4) - (6). In spite of the fact that these laws do not answer the usual Euclidean metric ρ_R any more, between any pair points on the directed axis of co-ordinates it is possible to measure distance by means of a usual ruler. We will show this property of the metric ρ_M with the help of fig. 2, 3. The first figure shows the distance between points $(\alpha^{-}; \beta^{+}) = (0{,}3^{-}; 0{,}8^{+})$ of the directed axis of co-ordinates when $M_{\alpha} = [0; 1]$, and the second - distance between points $(\alpha^{-}; \beta^{+}) = (-0{,}3^{-}; 0{,}8^{+})$ the directed axis of co-ordinates when $M_{\alpha} = [-1; 1]$.

Let's determine the distances between these points, using introduced in space of the directed levels of a membership the metric ρ_M.

For first two points we will receive (fig. 2):

$$\rho_M\,(\alpha^{-}; \beta^{+}) = |\,\alpha - \vee M_{\alpha}| + |\,\beta - \wedge M_{\alpha}| = \rho_M\,(0{,}3^{-}; 0{,}8^{+}) = |0{,}3 - 1\,| + |0{,}8 - 0\,| = 1{,}5.$$

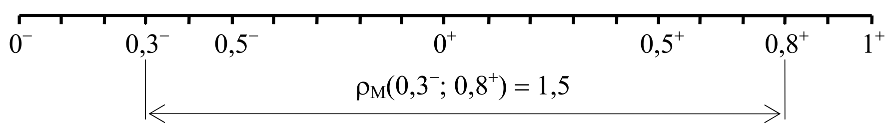

Fig. 2. Distance between the directed levels of a membership $0{,}3^{-}$ and $0{,}8^{+}$ at M_{α} [0; 1]

Let's find distance between the directed levels $-0{,}3^{-}$ and $0{,}8^{+}$ at $M_{\alpha} = [-1; 1]$ (fig. 3):

$$\rho_M\,(\alpha^{-}; \beta^{+}) = |\,\alpha - \vee M_{\alpha}| + |\,\beta - \wedge M_{\alpha}| = \rho_M\,(-0{,}3^{-}; 0{,}8^{+}) = |\,-0{,}3 - 1\,| + |0{,}8 - (-1)\,| = 3{,}1.$$

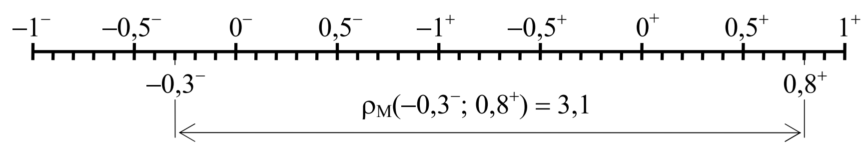

Fig. 3. Distance between the directed levels of a membership $-0{,}3^{-}$ and $0{,}8^{+}$ at $M_{\alpha} = [-1; 1]$

Noted property of the metric ρ_M allows to introduce definitions of continuous (discontinuous) and decreasing (increasing) functions of the directed levels of a membership by analogy to corresponding definitions of real-valued functions so, that continuous and discontinuous, as well as decreasing and increasing functions of the directed levels will look in the directed axes [5] similarly to corresponding real-valued functions in usual Cartesian axes.

Conclusions

On the set of the directed levels of weak sets there had been introduced the function which converts this set into metric space. It had been proved, that this function satisfies all the axioms of metric spaces, i.e. is the metric one. The suggested metric allows to measure distance between any points of an axis of the directed levels of a membership by means of a usual ruler, and allows to calculate the distance between any levels of a membership with an identical directions as usual

Euclidean distance between the corresponding not directed levels of a membership.

Such properties of the offered metric allow to introduce concepts of continuous and decreasing functions of levels of weak sets so that graphs of these functions in the directed axes of coordinates looked similarly to graphs of usual real-valued functions in Cartesian axes. It allows to greatly simplify the process of simulation and the analysis of indefinite parameters of complex systems by means of weak sets.

REFERENCES

1. Математичне моделювання процесів пошуку оптимальних рішень з використанням слабко заданих вхідних параметрів : матеріали сьомої міжнародної науково-технічної конференції “Контроль і управління в складних системах (КУСС – 2003)”, 8 –11 жовтня 2003 р., Вінниця / відп. ред. В. М. Дубовой. – Вінниця : УНІВЕРСУМ-Вінниця. – 2003. – С. 7 – 10.

2. Мокін Б. І. Слабкі множини та їх застосування до розв’язання задач прийняття рішень в умовах невизначеності даних / Б. І. Мокін, В. В. Камінський // Вісник Вінницького політехнічного інституту. – 2004. – №3. – С. 102-108.

3. Основы теории слабых множеств и её прикладные аспекты : материалы 12-й международной конференции по автоматическому управлению (Автоматика – 2005)”, 30 мая –3 июня 2005 г., Харьков. Т. 1 / науч. ред. Л. М. Любчик. – Харьков : Изд-во НТУ “ХПИ”. – 2005. – Т. 1. – С. 22 – 23.

4. Мокін Б. І. Математичне моделювання невизначених параметрів режиму електромереж з допомогою слабких множин / Б. І. Мокін, В. В. Камінський // Вісник Вінницького політехнічного інституту. – 2005. – №6. – С. 89 - 96.

5. Мокін Б. І. Геометрична інтерпретація слабких множин та їх систем нечітких реалізацій / Б. І. Мокін, В. В. Камінський // Вісник Вінницького політехнічного інституту. – 2006. – №4. – С. 34 - 47.

6. Мокін Б. І. Слабкі множини як альтернатива нечітким множинам в моделюванні невизначених параметрів складних систем / Б. І. Мокін, В. В. Камінський // Вісник Вінницького політехнічного інституту. – 2006. – №6. – С. 226-230.

7. Колмогоров А. Н. Элементы теории функций и функционального анализа : [учебн. для студ. матем. спец. универс.] / Колмогоров А. Н., Фомин С. В. – М. : Наука, 1981. – 744 с.

Boris Mokin – Professor of the Department for Modeling and Monitoring of Complex Systems;

Kaminsky Vyacheslav – Cand Sc. (Eng), Associate professor with the Department of Electrotechnical Systems of Power Consumption and Power Management.

Vinnytsia National Technical University.