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EQUIVALENCE OF INDETERMINED ALGORITHMS

Properties of algorithmic models, in particular, equivalence of indetermined algorithms are considered. The approach aimed at determination of equivalence degree of the algorithm in conditions of uncertainty by means of operator method.

Key words: *algorithmic models, equivalence of indetermined algorithms, uncertainty conditions, degree of equivalence.*

Enhancement of the efficiency of information systems (IS) becomes more and more actual problem due to globalization of all spheres of human life, complications and increase of IS volume. Growth of efficiency provides optimization of IS, distribution of the problems among subsystems of IS. etc. One of the promising directions aimed at the solution of this problem is optimization of IS algorithmic model with its further realization by hardware – software means [1, 2].

Optimization of algorithmic model (AM) is realized with the help of the system of equivalent transformations [3]. Equivalent transformations are transformations as a result of which we obtain algorithmic model. Two algorithmic models are called functionally equivalent, if at identical input data, they produce identical results.

Considerable contribution in the study of algorithms equivalence was made by A. A. Liapunov, who introduced the notion of programmes schemes [4]. On the basis of algorithms standard schemes principle notions and properties, connected with algorithmic models are introduced, principle notion being relation of functional equivalence of algorithmic models.

Ideas of Liapunov were further developed at the end of 50'th and 60'th by A. P. Ershov, N. A. Krinitskiy, L. A. Kaluzhnin, R. I. Podlovchenko and Yu. I. Yanov, who is [5] formalized the notion of programme scheme, defined relation of schemes equivalence and studied the problem of equivalence for the class of schemes, which later were called Yanov's schemes.

N. A. Krinitskiy [6] investigated the problem of equivalence and equivalent transformations of standard schemes, namely, for the subclass of schemes without cycles (i. e, schemes, the graph of which does not contain contours), he found the algorithm the equivalence recognition, thus the complete system of transformations has been elaborated, which allows to transform automatically any pair of equivalent schemes into one to one. Graph form of the schemes was suggested by Kaluzhnin [7].

Algorithmic models of information systems were considered in determined conditions [4 - 8]. But in greater part of practical problems formation of IS occurs in conditions of uncertainty (CU) of the input data, the degree and origin of uncertainty can differ greatly, in particular, its reasons can be: scarce knowledge of application field, lack of accurate information about the value of the data, uncertainty of the aim, etc.

Account of uncertainty opens new possibilities in the sphere of design and optimization of IS on the basis of algorithmic models. Let two information systems IS_1 and IS_2 be, system IS_2 being less expense variant of the system IS_1 of the same designed $C_1 > C_2$, where C_1, C_2 - expenditures of corresponding information system IS_1 and IS_2.

Algorithmic models AM_1 and AM_2 which define transformation of input data X on the result of functioning Y correspond to these systems. Since system models differ, then the results of functioning will be different, hence:

$$Y_1 = AM_1(X) \text{ и } Y_2 = AM_2(X).$$

Let us characterize the results Y_1 and Y_2 by uncertainty functions $\beta_1(Y_1)$ and $\beta_2(Y_2)$ correspondingly, shown in Fig 1. Validity of coincidence of the results in conditions of uncertainty

$$B(Y_1 = Y_2) = \int_{\Omega_{Y_1} \cap \Omega_{Y_2}} \beta_1(Y)\beta_1(Y)dY,$$

where Ω_{Y_1} and Ω_{Y_2} – are areas of results values Y_1 and Y_2 correspondingly.

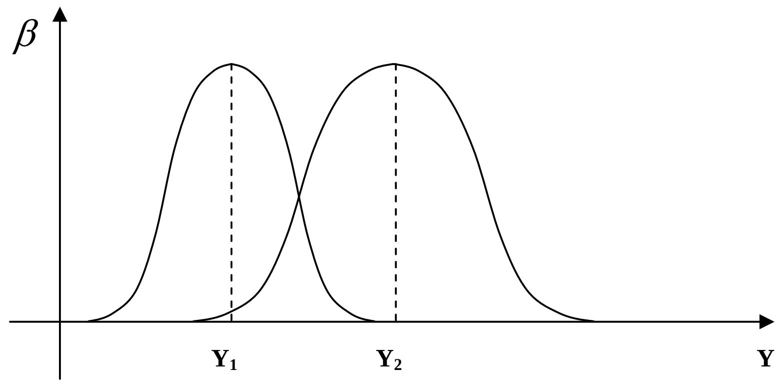


Fig. 1. Uncertainty functions of the results Y1 and Y_2

Proceeding from the definition of algorithmic models equivalence, we may state, that systems IS_1 and IS_2 in conditions of uncertainty are equivalent with the validity $B(Y_1 = Y_2)$. Then, the choice of efficient variant of IS is reduced to evaluation of risk value $Q = (1 - B)/(C_1 - C_2)$.

Thus, for development of information systems the problem of investigation of algorithmic models equivalence in conditions of uncertainty is very actual.

The task of determination of algorithmic models equivalence in conditions of uncertainty does not have generally – accepted approaches to its solution. Improvement of the methods of analysis of algorithmic models equivalence to take into account the conditions of uncertainty of systems functioning is the problem to be solved in this paper.

The class of standard schemes is characterized by the basis of class *B* and the structure of the scheme. Basis of the class fixes symbols, the schemes are constructed of, defines their role (variables, functional symbols, etc) sets the form of expression of operators of the schemes [8]. Fixation of the interpretation turns standard scheme into certain algorithmic model. Interpretation the basis *B* in the area of interpretation *D* is called function *I,* which compares each element (variables, functional symbols, predicates) from the basis *B,* with certain defined functions and elements from the area of interpretation D. The pair (*S, I*), where *S* is the scheme in the basis *B*, and *I* is interpretation of this basis, is called interpreted standard scheme of the algorithm or algorithmic model.

In [8] the relation of equivalence for standard schemes of algorithms in one basis is introduced.

If schemes S_1 and S_2 are constructed in two different bases B_1 and B_2, then they can be "reduced to one basis", union of bases B_1 and B_2. Standard schemes S_1 and S_2 in the basis *B* are functionally equivalent ($S_1 \sim S_2$), if for any interpretation *I* of the basis *B* programme (S_1, *I*) and (S_2, *I*) or both are recycled, or both stop with the same result, i. e. $val(S_1, I) \approx val(S_s, I)$. Equivalent transformations of algorithmic model we will call such series of operations over the model, which is not change the content o9f system operation results.

Notions, introduced allow to pass to definition of algorithmic models equivalence in conditions of uncertainty.

In conditions of uncertainty this notion has bluring boundaries. Taking into account limited validity of algorithm operation result, obtained in conditions of uncertainty, we may speak only about the equivalence of the algorithms with preset validity or about the degree of algorithmic models equivalence. Let us consider the possible approach to the evaluation of the validity.

Uncertainty can be described by different methods. Let us make use the functional method of description. While functional method, uncertainty of stochastic type is described by probability distributions, and uncertainty of fuzzy type is described by property functions. Method of generalized functions [9] takes into account uncertainty of various type.

By generalized function we mean positively defined function in the interval of possible values of the argument, which is designated by $\beta(x)$ and characterizes the possibility of π or probability

P acceptance by the argument the value from the definite interval [x_1, x_2], $x_1 \in B$, $x_2 \in B$, according to rules:

$$p = \frac{\int_{x_1}^{x_2} d[\beta(x)]}{\int_B d[\beta(x)]}, \qquad \pi = \frac{\int_{x_1}^{x_2} d[\beta(x)]}{\max\limits_B \int_{[x_{i-1}, x_i]} d[\beta(x)]}, \tag{1}$$

where $x_{i-1}, x_i \subset B$, $i = \overline{1,n}$, n – is the number of intervals of B division.

For generalized function rules of mathematical operations generalization are divided into three groups: non- linear unary, non – linear binary, integer – differential. Operator method of transformation, which uses integral operators is taken as basis for definition of these operations:

$$\beta_Y(y) = \int_{-\infty}^{+\infty} \overset{n}{\cdots} \int_{-\infty}^{+\infty} \beta_X(\bar{x}) \psi(\bar{x}, y, F, Q) \mathrm{d}\bar{x}, \tag{2}$$

where ψ – is kernal of operator, F and Q – are characteristics of the operation, being performed; n – is integration multipleness, which depends on vector dimensionality $\bar{x}$ and characteristics of the operation F and Q.

Functions of uncertainty of two algorithms operation results, the degree of equivalence of which is investigated, can be obtained by means of operator method. For this purpose algorithms are presented in algebraic form with further transformation into operator form of recording. For transformation of determined model (*R* - form) of operators $\tilde{u}$ into generalized (*G* - form) initial model is written in the form of series of symbols, which form certain mathematical formula in system *R*.

While writing the model indefinite changing, operation signs {+, -, *, /}, signs of elementary functions, designation of integral – differential (dynamic) transformation in the form of Duhamel integral *I*(*x***g*), where *x* – is initial function, *g* – is kernel of transformation (pulse transition function of dynamic transformation), dividers are used.

Examples of typical notations are given in Table 1

Transformation of algebraic model is formalized in the form of algebraic system [3]. Equivalent transformations of algorithmic model are realized on the basis of properties:

- *paste(B,n1,n2) cut(n1,n2) ≡ 1;*
- *cut(n1,n2) paste(B,n1,n2) ≡ 1;*
- *paste(B1,n1,n2) paste(B2,n3,n4) ≡ paste(B2,n3,n4) paste(B1,n1,n2), если (n1,n2) ∩ (n3,n4) = ∅;*
- *cut(n1,n2) cut(n3,n4) ≡ cut(n3,n4) cut(n1,n2), если (n1,n2) ∩ (n3,n4) = ∅;*
- *En1(op,X,Y) En2(op-1,Y,X) ≡ 1.*

In [9] ratios of indeterminated data comparison in system G are defined:

Definition 1. Indeterminated data *x, y* are assumed to be equal считаются равными $X = Y$ if $\beta_X = \beta_Y$.

Definition 2. For indeterminated data $X \underset{\sim}{\geq} Y$, if $Z = X - Y$ and

$$\int_0^{+\infty} \beta_Z dz > \int_{-\infty}^0 \beta_Z dz.$$

For evaluation of equivalence of algorithms in condition uncertainty we will introduce the notion equality degree.

Table 1

Examples of typical notations of algorithmic model in algebraic and operator forms

Notation in algorithmic language	Algebraic form (R-form)	Comment	Operator form (G-form)	Comment
Checking of condition and branching if (*a*) then *n1* else *n2*	<<$E_1(n_1)$ w2 (i>a)_|1 $E_2(n_2)$>>	*a* – logic variable.; $n1 \in N$ – number of algorithm operator, to which the transmission is performed in case of valid *a*, $n2 \in N$ – in case of false; *N* – numbered set of algorithm operators	Operator model will be have the form of two – component vector $b(g)=\left\{\begin{matrix} b(a1)\cdot(n1), \\ b(a2)\cdot b(n2) \end{matrix}\right\}$	$b(a)=k\cdot\delta[a-a1]+$ $+(k-1)\cdot\delta[a-a2],$ *k* – probability of true value of condition *a*
Computation of function $p_2=f(p_1)$	I($f(p_1)$ / p_2)	p_1 – initial data; p_2 – result of computation ; *f* – formula of computation	b(p2)=F (1, f)[b(p1)]	*F* (1, *f*) – operator *n*-th order; non - linear binary operation; $b(p_1)$,$b(p_2)$ – generalized functions
Constants initialization # define p_2, p_1	C(p_1 /p_2)	p_1 – value of constant; p_2 – denomination of constant	$b(p_1)=\delta[p_1]$ $b(p_2)=F(1,1)[b(p_1)]$	$\delta[p2-p1]=$ $=\begin{vmatrix} 0, if\ \ p1\neq p_2 \\ \infty, if\ \ p_1=p_2 \end{vmatrix}$ $\int\limits_{-\infty}^{+\infty}\delta[p_2-p_1]dp_2=1$
Measurement import (&p_1) p_2=f(p_1) $p_3=\varepsilon$	Im(p_1,ε /p_2,p_3)	p_1 – value being measured; p_2 – measurement result ; p_3 – error of measurement	$b(p_2)=F(2,+)[b(p_1),b(p_2)]$	$b(p_3)=\frac{1}{2\pi p_3}e^{-\frac{(p_3)^2}{2\varepsilon^2}}$ $b(p_2)=\delta[p_1]$ Normal distribution of measurement errors is possible
Expert data scan (&p_1, &p_2); $p_3=(p_1+p_2)/2$; $p_4=(p_1-p_2)/6$	Ex(p_1,p_2 / p_3, p_4)	p_1, p_2 – left and right boundaries of experts evaluation ;	$b(p_3)=F(1, N)[e(p)]$	$e(p)=\begin{vmatrix} 1, -0.5\le p\le +0.5 \\ 0, \lvert p\rvert > 0.5 \end{vmatrix}$ $N=p\cdot(p_2-p_1)+\frac{p_1+p_2}{2}$
Delay (τ)	I (p1(t- τ) / p2(t))	τ – time of delay	$b(p_2)=F(n,g_\tau)[b(p_1)]$	g_τ – pulse transition function of delay link $g_\tau=\frac{1}{2\pi}\int\limits_0^\infty e^{-p\tau}e^{pt}dp$
Start (end) of the cycle { }	A(B), A(E)			

Definition 3. The degree of equality of indetermined data *x* and *y*, which are characterized by uncertainty functions β_x and β_y, we will call value

$$d=\int\limits_{-\infty}^{+\infty}\beta_{xy}(x=\xi,y=\xi)d\xi=\int\limits_{-\infty}^{+\infty}\beta_x(\xi)\cdot\beta_y(\xi)d\xi\,. \tag{4}$$

It is obviously, that *d=1* if $x=y$ by *definition 1*.

The result of algorithm operation can be defined on the basis of such types of representation:

1) numerical value, which belongs to certain continuous interval of possible values;
2) numerical value, which belongs to certain finite discrete set of possible values;
3) action, which is realized by other engineering facilities of the system;
4) image on the screen.

For application of the *Definition 3* for each type of the result it is necessary to define metric with the account of uncertainty. Taking into account the convenience of application for solution of optimization problems, we use Euclide metric.

Metric of the results of the first type we define according to the expression:

$$M_1 = \int_{-\infty}^{+\infty} z^2 \cdot \beta(z) \cdot dz \,, \tag{5}$$

where $z = x - y$.

Taking into account the independence of the results of two algorithms, we obtain:

$$M_1 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x-y)^2 \cdot \beta(x)\beta(y) \cdot dx \cdot dy \,.$$

Metric of the results of the second type we define according to the expression:

$$M_2 = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - y_i)^2 \cdot \mathrm{B}_{x_i} \cdot \mathrm{B}_{y_j} \,, \tag{6}$$

where n – is the power of the set of results; $\mathrm{B}_\xi : \beta_\xi = \sum_{i=1}^{n} \mathrm{B}_{\xi_i} \cdot \delta[\xi - \xi_i]$; $\delta[\xi]$ – delta - function.

Metric of the results of the third type we define according to the expression (6), assuming that $\{y\}$ – is the set of possible actions of engineering facilities of the system.

Metric of the results of the fourth type is introduced separately for two cases: when the image on the screen is selected from certain set of standard images, then definition of matric is analogous to the expression (6), and when the image has permanent structure with variable parameters (for instance, graph of the function) then definition if the matric is analogous to the expression (5), where *x* and *y* are sets of indetermined parameters of the images of two algorithms.

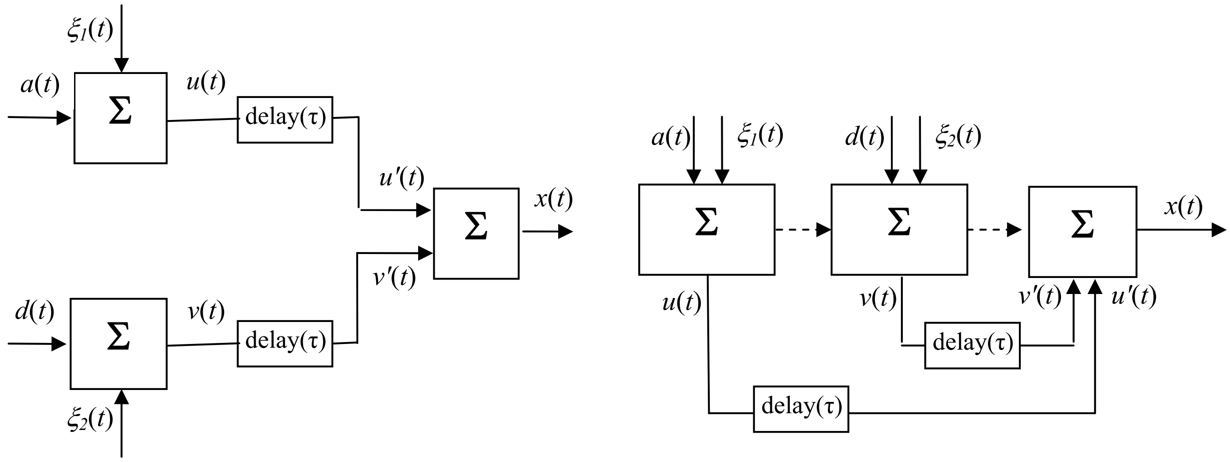
Let us consider the example. The system of signals processing, shown in Fig 2, can be realized both in parallel (2, a) and serial (2, b) versions.

Algorithmic model of parallel execution is of the following form:

M1=A(B) || [Im_1(a(t) / $A\sin[\omega(t+\tau)]$, ε1) I_2(f(a(t),ξ_1) / u(t)=a(t)+ξ_1) I_3(u(t) / u'(t+τ)) Im_4(d(t) / $D\sin[\omega(t+\tau)]$,ε2) I_5 (f(d(t),ξ_2) / v(t)=d(t)+ξ_2) I_6(v(t) / v'(t+τ))] I_7(f(u'(t+τ),v'(t+τ)) / x(t)) A(E). (7)

Algorithmic model of serial execution in algebraic form is:

M2=A(B) Im_1(a(t) / $A\sin[\omega(t+\tau)]$, ε1) I_2(f(a(t),ξ_1)/u(t)=a(t)+ξ_1) I_3(u(t)/u'(t+τ)) Im_4(d(t+τ) / $D\sin[\omega(t+2\tau)]$,ε3) I_5 (f(d(t+ τ),ξ_2) / v(t+ τ)=d(t +τ)+ξ_2) I_6(v(t+ τ) / v'(t+2τ)) I_7(f(u'(t+ τ),v'(t+2 τ)) / x(t)) A(E). (8)



а) б)

Fig. 2. Example of signals processing system

Algorithmic models (7) and (8in operator form have the following form:

$b_1(x) = F_7(2,+)[\ F_6(n,g_\tau)\ [F_5(2,+)[\beta(\xi_2)\ ,\ F_4(2,+)[b(d),\ b(\varepsilon_2)]]]\ ,\ F_3(n,\ g_\tau)[F_2(2,+)[b(\xi_1),\ F_1(2,+)[b(a),\ b(\varepsilon_1)]],$

$b_2(x) = F_7(2,+)[\ F_6(n,g_{2\tau})\ [F_5(2,+)[\beta(\xi_2)\ ,\ F_4(2,+)[b(d),\ b(\varepsilon_2)]]]\ ,\ F_3(n,\ g_\tau)[F_2(2,+)[b(\xi_1),\ F_1(2,+)[b(a),\ b(\varepsilon_1)]].$

Let signals, arriving at the inputs of the algorithms be:

$$a(t) = A\sin\omega t\,,\ \ d(t) = D\sin\omega t\,.$$

White normal noise ξ_1 and ξ_2 arrives at the inputs.

In conditions of definiteness (if there is non noise) the result is obtained by means of *R*-model. For the schemes of Fig. 2, a, b we obtain:

a) $A\sin[\omega(t+\tau)] + D\sin[\omega(t+\tau)] = (A+D)\sin[\omega(t+\tau)]$,

this implies $Y_1 = (A+D)$.

б) $A\sin[\omega(t+\tau)] + B\sin[\omega(t+2\tau)] = \sqrt{A^2 + 2AB\cos(\omega\tau) + B^2}\,\sin[\omega t + \phi]$,

this implies $Y_2 = \sqrt{A^2 + 2AD\cos(\omega\tau) + D^2}$.

It is obviously, that in the conditions of definiteness algorithmic models (7) and (8) are not equivalent. In conditions of uncertainty (if noise is available) the result of transformation will be obtained by means of *G* –model, which enables to define functions of uncertainty results

$\beta(Y_1^*)$ и $\beta(Y_2^*)$:

a) $Y_1^* = (A+D) + \xi_1 + \xi_2$;

b) $Y_2^* = \sqrt{A^2 + 2AD\cos(\omega\tau) + D^2} + \xi_1 + \xi_2$;

In conditions of normal white noise functions of uncertainty results will be Gaussians with average values Y_1 and Y_2 correspondingly and dispersions $D_{Y^*} = \sigma^2_{\xi_1} + \sigma^2_{\xi_2}$.

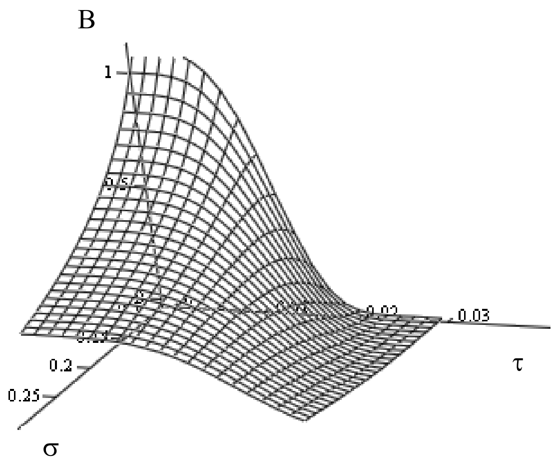
Validity of results coincidence in conditions of uncertainty:

:

$$B(Y_1^* = Y_2^*) = \int\limits_{-(A+B)}^{+(A+B)} \left[\frac{1}{2\pi\sqrt{\sigma^2_{\xi_1} + \sigma^2_{\xi_2}}} e^{-\frac{(Y-Y_1)^2}{2(\sigma^2_{\xi_1} + \sigma^2_{\xi_2})}}\right]\left[\frac{1}{2\pi\sqrt{\sigma^2_{\xi_1} + \sigma^2_{\xi_2}}} e^{-\frac{(Y-Y_2)^2}{2(\sigma^2_{\xi_1} + \sigma^2_{\xi_2})}}\right] dY = \frac{1}{2\sqrt{2}\pi\cdot\sigma} e^{-\left(\frac{y_1-y_2}{2\sigma}\right)^2}.$$

Dependence of models (7) and (8) equivalence degree in conditions of uncertainty on the time of operation execution and noise dispersion is shown in Fig 3. It is seen in Fig 3, that the validity of results coincidence for the systems, shown in Fig 2a and 2b will be maximum at sum of noise dispersion $\sigma = 0.1$ if parameters of signals and operation rate of the blocks, taken for the sake of example, is $0.15 < \tau < 0.25$.

It follows from the analysis of models equivalence that at certain conditions serial system that can be realized with less number of hardware facilities, is equivalent to parallel system to degree of equivalence can be increased by means of artificial introduction of uncertainty.



A=1
D=1
ω=2×π×10

Fig. 3. Dependence of models equivalence degree in conditions of uncertainty on noise dispersion

Conclusions

For evaluation of algorithms equivalence degree in conditions of uncertainty the notion of equality degree is introduced. Uncertainty functions of the results of two algorithms operation, degree of equivalence of which is investigated, are obtained by means of operator method. It is proved that algorithms which are not equivalent in conditions can be equivalent in indefinite conditions, equivalence degree can be increased by artificial introduction of uncertainty.

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