I. V. Bogach, Cand. Sc. (Eng); M. A. Mashnytskyy; A. F. Homchuk

SIMULATION OF THREE DEMENTIONAL SURFACES ON THE BASIS OF MODIFICATION OF DIFFERENTIAL BESSELIAN METHOD

The paper presents the developed method for simulation of three-dimensional surfaces on the basis of modification of differential Besselian method. There had been considered an example of application of the developed method.

Key words: *interpolation, surface simulation , differential Besselian method, function of two variables, three-dimensional surface.*

Task setting

Simulation of three-dimensional surfaces is rather actual task. Rapid development of equipment, electronic systems causes the necessity in researching the efficiency of the developed devices, which depend on coordinates of its location, for example, humidity changes in the room, which depends on air conditioning placement. During the research of the object which is described by the function of three variables there appears a task of identification of its mathematical model. This task is appropriate to be solved by mathematical simulation which combines experiment and theory. Such simulation is used in medicine, space researches, geophysics, which expands the sphere for introduction the research results apart from traditional computer graphics.

One of the widespread methods of function simulation is interpolation Besselian method, developed only for function of two variables. The paper suggests the modification of the Besselian method for simulation of three dimensional surfaces.

The task of simulation of three-dimensional surfaces on the basis of function interpolation is described on building function $F(x, y, z)$, which accepts in some points x_i, y_j, z_k $(i, j, k = \overline{0, n})$, which are called nodes of interpolation (separate values will be written $F(x_0, y_0, z_0) = \varphi_{000}$, $F(x_1, y_1, z_1) = \varphi_{111}$, …, $F(x_n, y_n, z_n) = \varphi_{nnn}$). In general case the interpolation of function is reduced to finding its non-table values [1 – 5].

The task of simulation of three-dimensional surfaces are to be solved in cases of image processing, topography information, in different kinds of diagnosing etc. Leading organization, elaborating graphics accelerators and software for three-dimensional simulation and creation of special effects conduct researches in this direction.

The objective of this work is to improve efficiency of simulation by widening of classical methods using three variables.

Description of method

To deduce a Besselian formula we use the second interpolation Gauss formula [6]:

$$\begin{aligned} P(x,y,z) &= \varphi_{0,0,0} + p\Delta^{100}\varphi_{-1,0,0} + q\Delta^{010}\varphi_{0,-1,0} + r\Delta^{001}\varphi_{0,0,-1} + \frac{(p+1)p}{2!}\Delta^{200}\varphi_{-1,0,0} + \\ &+ pq\Delta^{110}\varphi_{-1,-1,0} + pr\Delta^{101}\varphi_{-1,0,-1} + qr\Delta^{011}\varphi_{0,-1,-1} + \frac{(q+1)q}{2!}\Delta^{020}\varphi_{0,-1,0} + \\ &+ \frac{(r+1)r}{2!}\Delta^{002}\varphi_{0,0,-1} + \frac{(p+1)p(p-1)}{3!}\Delta^{300}\varphi_{-2,0,0} + \frac{(q+1)q(q-1)}{3!}\times \\ &\times\Delta^{030}\varphi_{0,-2,0} + \frac{(r+1)r(r-1)}{3!}\Delta^{003}\varphi_{0,0,-2} + \frac{(p+1)pq}{2!}\Delta^{210}\varphi_{-1,-1,0} + \\ &+ \frac{(p+1)pr}{2!}\Delta^{201}\varphi_{-1,0,-1} + \frac{p(q+1)q}{2!}\Delta^{120}\varphi_{-1,-1,0} + \frac{(q+1)qr}{2!}\Delta^{021}\varphi_{0,-1,-1} + \\ &+ \frac{p(r+1)r}{2!}\Delta^{102}\varphi_{-1,0,-1} + \frac{q(r+1)r}{2!}\Delta^{012}\varphi_{0,-1,-1} + pqr\Delta^{111}\varphi_{-1,-1,-1} + \ldots \end{aligned} \qquad (1)$$

Lets take $(2n+2)^3$ equidistant nodes of interpolation in the direction of all variables of interpolating: $(x_{-n}, y_{-n}, z_{-n}), (x_{-(n-1)}, y_{-(n-1)}, z_{-(n-1)}), \ldots, (x_0, y_0, z_0), \ldots, (x_{n-1}, y_{n-1}, z_{n-1}), (x_n, y_n, z_n)$, $(x_{n+1}, y_{n+1}, z_{n+1})$ with step h_x, h_y, h_z correspondingly, to the direction of interpolagion $\varphi_{i,j,k} = f(x_i, y_j, z_k)$, $(i = \overline{-n, n+1},\ j = \overline{-n, n+1},\ k = \overline{-n, n+1})$ – set values of function $\varphi = f(x,y,z)$

If we accept as the initial value $x = x_0,\ y = y_0,\ z = z_0$ and $\varphi = \varphi_0$, and use the nodes $(x_i, y_j, z_k),\ (i = \overline{-n, n},\ j = \overline{-n, n}, k = \overline{-n, n})$, we receive an expression:

$$\begin{aligned} P(x,y,z) &= \varphi_{0,0,0} + p\Delta^{100}\varphi_{-1,0,0} + q\Delta^{010}\varphi_{0,-1,0} + r\Delta^{001}\varphi_{0,0,-1} + \frac{(p+1)p}{2!}\Delta^{200}\varphi_{-1,0,0} + \\ &+ pq\Delta^{110}\varphi_{-1,-1,0} + pr\Delta^{101}\varphi_{-1,0,-1} + qr\Delta^{011}\varphi_{0,-1,-1} + \frac{(q+1)q}{2!}\Delta^{020}\varphi_{0,-1,0} + \\ &+ \frac{(r+1)r}{2!}\Delta^{002}\varphi_{0,0,-1} + \frac{(p+1)p(p-1)}{3!}\Delta^{300}\varphi_{-2,0,0} + \frac{(q+1)q(q-1)}{3!}\times \\ &\times\Delta^{030}\varphi_{0,-2,0} + \frac{(r+1)r(r-1)}{3!}\Delta^{003}\varphi_{0,0,-2} + \frac{(p+1)pq}{2!}\Delta^{210}\varphi_{-1,-1,0} + \\ &+ \frac{(p+1)pr}{2!}\Delta^{201}\varphi_{-1,0,-1} + \frac{p(q+1)q}{2!}\Delta^{120}\varphi_{-1,-1,0} + \frac{(q+1)qr}{2!}\Delta^{021}\varphi_{0,-1,-1} + \\ &+ \frac{p(r+1)r}{2!}\Delta^{102}\varphi_{-1,0,-1} + \frac{q(r+1)r}{2!}\Delta^{012}\varphi_{0,-1,-1} + pqr\Delta^{111}\varphi_{-1,-1,-1} + \ldots \end{aligned} \qquad (2)$$

We accept as the initial values $x = x_1,\ y = y_1,\ z = z_1$ and $\varphi = \varphi_1$, and use the nodes $(x_{1+i}, y_{1+j}, z_{1+k}),\ (i = \overline{-n, n},\ j = \overline{-n, n}, k = \overline{-n, n})$.Than

$$\frac{x - x_1}{h_x} = \frac{x - x_0 - h}{h_x} = p - 1; \quad \frac{y - y_1}{h_y} = \frac{y - y_0 - h}{h_y} = q - 1; \quad \frac{z - z_1}{h_z} = \frac{z - z_0 - h}{h_z} = r - 1,$$

and, correspondingly, indexes of all differences in the right part of the formula (2) increase by unit. If in the right part of formula we change (2) *q* by *q – 1* increase index of all differences by 1, we receive auxiliary interpolation formula:

$$
\begin{aligned}
P(x,y,z) = {} & \varphi_{1,1,1} + (p-1)\Delta^{100}\varphi_{0,1,1} + (q-1)\Delta^{010}\varphi_{1,0,1} + (r-1)\Delta^{001}\varphi_{1,1,0} + \frac{p(p-1)}{2!}\times \\
& \times \Delta^{200}\varphi_{0,1,1} + (p-1)(q-1)\Delta^{110}\varphi_{0,0,1} + (p-1)(r-1)\Delta^{101}\varphi_{0,1,0} + 3 \\
& + (q-1)(r-1)\Delta^{011}\varphi_{1,0,0} + \frac{q(q-1)}{2!}\Delta^{020}\varphi_{1,0,1} + \frac{r(r-1)}{2!}\Delta^{002}\varphi_{1,1,0} + \\
& + \frac{p(p-1)(p-2)}{3!}\Delta^{300}\varphi_{-1,1,1} + \frac{q(q-1)(q-2)}{3!}\Delta^{030}\varphi_{1,-1,1} + \\
& + \frac{r(r-1)(r-2)}{3!}\Delta^{003}\varphi_{1,1,-1} + \frac{p(p-1)(q-1)}{2!}\Delta^{210}\varphi_{0,0,1} + \\
& + \frac{p(p-1)(r-1)}{2!}\Delta^{201}\varphi_{0,1,0} + \frac{(p-1)q(q-1)}{2!}\Delta^{120}\varphi_{0,0,1} + \\
& + \frac{q(q-1)(r-1)}{2!}\Delta^{021}\varphi_{1,0,0} + \frac{(p-1)r(r-1)}{2!}\Delta^{102}\varphi_{0,1,0} + \\
& + \frac{(q-1)r(r-1)}{2!}\Delta^{012}\varphi_{1,0,0} + (p-1)(q-1)(r-1)\Delta^{111}\varphi_{0,0,0} + \ldots
\end{aligned}
\tag{3}
$$

We take the arithmetic mean of the formulas (2) and (3) and after not difficult transformations we receive interpolation Besselian formula.

$$
\begin{aligned}
P(x,y,z) = {} & \frac{\varphi_{1,1,1} + \varphi_{0,0,0}}{2} + (p-\frac{1}{2})\Delta^{100}\varphi_{0,1,1} + (q-\frac{1}{2})\Delta^{010}\varphi_{1,0,1} + (r-\frac{1}{2})\Delta^{001}\varphi_{1,1,0} + \\
& + \frac{p(p-1)}{2!}\frac{\Delta^{200}\varphi_{0,1,1} + \Delta^{200}\varphi_{-1,0,0}}{2} + (p-\frac{1}{2})(q-\frac{1}{2})\Delta^{110}\varphi_{0,0,1} + (p-\frac{1}{2})\times \\
& \times (r-\frac{1}{2})\Delta^{101}\varphi_{0,1,0} + 3(q-\frac{1}{2})(r-\frac{1}{2})\Delta^{011}\varphi_{1,0,0} + \frac{q(q-1)}{2!}\times \\
& \times \frac{\Delta^{020}\varphi_{1,0,1} + \Delta^{020}\varphi_{0,-1,0}}{2} + \frac{r(r-1)}{2!}\frac{\Delta^{002}\varphi_{1,1,0} + \Delta^{002}\varphi_{0,0,-1}}{2} + \\
& + \frac{p(p-\frac{1}{2})(p-1)}{3!}\Delta^{300}\varphi_{-1,1,1} + \frac{q(q-\frac{1}{2})(q-1)}{3!}\Delta^{030}\varphi_{1,-1,1} + \frac{r(r-\frac{1}{2})(r-1)}{3!}\times \\
& \times \Delta^{003}\varphi_{1,1,-1} + \frac{p(p-1)(q-\frac{1}{2})}{2!}\frac{\Delta^{210}\varphi_{0,0,1} + \Delta^{210}\varphi_{-1,-1,0}}{2} + \frac{p(p-1)(r-\frac{1}{2})}{2!}\times \\
& \times \frac{\Delta^{201}\varphi_{0,1,0} + \Delta^{201}\varphi_{-1,0,-1}}{2} + \frac{(p-\frac{1}{2})q(q-1)}{2!}\frac{\Delta^{120}\varphi_{0,0,1} + \Delta^{120}\varphi_{-1,-1,0}}{2} + \\
& + \frac{q(q-1)(r-\frac{1}{2})}{2!}\frac{\Delta^{021}\varphi_{1,0,0} + \Delta^{021}\varphi_{0,-1,-1}}{2} + \frac{(p-\frac{1}{2})r(r-1)}{2!}\times \\
& \times \frac{\Delta^{102}\varphi_{0,1,0} + \Delta^{102}\varphi_{-1,0,-1}}{2} + \frac{(q-\frac{1}{2})r(r-1)}{2!}\frac{\Delta^{012}\varphi_{1,0,0} + \Delta^{012}\varphi_{0,-1,-1}}{2} + \\
& + (p-\frac{1}{2})(q-\frac{1}{2})(r-\frac{1}{2})\Delta^{111}\varphi_{0,0,0} + \ldots,
\end{aligned}
\tag{4}
$$

where $p=\dfrac{x-x_0}{h_x}$, $q=\dfrac{y-y_0}{h_y}$, $r=\dfrac{z-z_0}{h_z}$.

We introduce designation of generalized degree for the function of three variable. Generalized degree of numbers p, q and r we will name multiplicand, which contain n multiplicands, first of which equals $\left(p-\frac{1}{2}\right)$, $\left(q-\frac{1}{2}\right)$ and $\left(r-\frac{1}{2}\right)$, on condition that n – unpaired number. If n – unpaired number, the first multiplicand equals p, q, and r, and each following is i^2 less than the previous ($i=\overline{1..(\lceil n/2\rceil-1)}$):

$$
\begin{aligned}
p^{[n]} &= \left(p-\frac{1}{2}\right)^{c} p(p-1)(p+1)(p-2)(p+2)\ldots(p+(w-1));\\
q^{[n]} &= \left(q-\frac{1}{2}\right)^{c} q(q-1)(q+1)(q-2)(q+2)\ldots(q+(w-1));\\
r^{[n]} &= \left(r-\frac{1}{2}\right)^{c} r(r-1)(r+1)(r-2)(r+2)\ldots(r+(w-1)),
\end{aligned}
\tag{5}
$$

where $c=\begin{cases}1, \text{при } n=1,3,5,\ldots\\ 0, \text{при } n=2,4,6,\ldots\end{cases}$; $w=\lceil n/2\rceil$.

$$
P(x,y,z)=\sum_{i=0}^{2n}\sum_{j=0}^{2n}\sum_{k=0}^{2n}\frac{p^{[i]}\cdot q^{[j]}\cdot r^{[k]}}{i!\cdot j!\cdot k!}\,\frac{\Delta^{i,j,k}\varphi_{m,l,u}+\Delta^{i,j,k}\varphi_{m+1,l+1,u+1}}{2}
\tag{6}
$$

где $m=-\lceil i/2\rceil$; $l=-\lceil j/2\rceil$; $u=-\lceil k/2\rceil$.

interpolation Besselian formula (6), as follows from deducing is a polynomial which coincides with the set function $\varphi=f(x,y,z)$ in *2n+2* points $(x_{-n},y_{-n},z_{-n}),(x_{-(n-1)},y_{-(n-1)},z_{-(n-1)}),\ldots,(x_0,y_0,z_0),\ldots,(x_{n-1},y_{n-1},z_{n-1}),(x_n,y_n,z_n),(x_{n+1},y_{n+1},z_{n+1})$.

Evaluation of error of interpolation Besselian formula

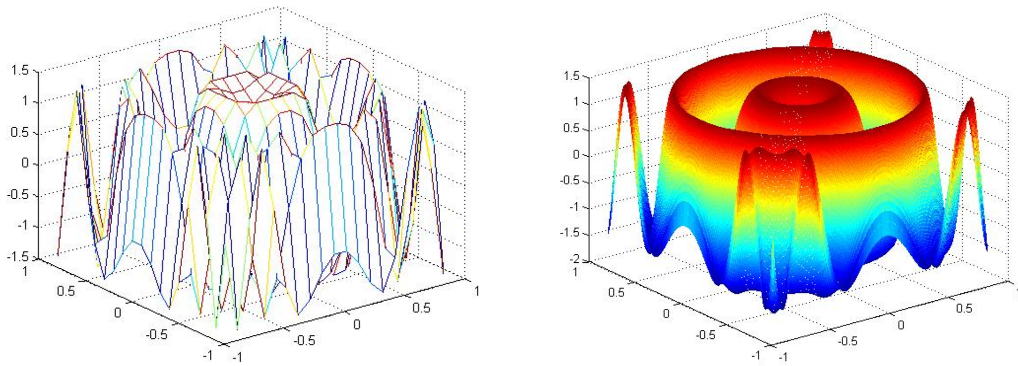
If *2n+1* – order of maximum used difference of the table, $x\in[x_0-nh,x_0+nh]$, $y\in[y_0-nh,y_0+nh]$ та $z\in[z_0-nh,z_0+nh]$ then:

$$
\begin{aligned}
R_n(x,y,z) &= q(q^2-1^2)(q^2-2^2)\ldots(q^2-(n+1))\,p(p^2-1^2)(p^2-2^2)\ldots(p^2-(n+1))\,r(r^2-1^2)\times\\
&\times(r^2-2^2)\ldots(r^2-(n+1))\frac{h^{2n+2}f^{(2n+2),(2n+2),(2n+2)}(\xi_x,\xi_y,\xi_z)}{(2n+2)!(2n+2)!(2n+2)!},
\end{aligned}
\tag{7}
$$

where $p=\dfrac{x-x_0}{h_x}$, $q=\dfrac{y-y_0}{h_y}$, $r=\dfrac{z-z_0}{h_z}$, $\xi_x\in[x_0-nh,x_0+nh]$, $\xi_y\in[y_0-nh,y_0+nh]$ and $\xi_z\in[z_0-nh,z_0+nh]$.

If function $f(x,y,z)$ is set in the kind of table, then with the small step of h there shall be accepted:

$$R_n(x,y,z) = q(q^2-1^2)(q^2-2^2)\ldots(q^2-n^2)p(p^2-1^2)(p^2-2^2)\ldots(p^2-n^2)\times \\ \times r(r^2-1^2)(r^2-2^2)\ldots(r^2-n^2)\times \\ \times \frac{\Delta^{(2n+1),(2n+1),(2n+1)}y_{-n-1,-n-1,-n-1} + \Delta^{(2n+1),(2n+1),(2n+1)}y_{-n,-n,-n}}{2(2n+1)!(2n+1)!(2n+1)!}, \quad (8)$$



а) б)

Fig. 1. Building surfaces by Besselian method

Fig. 1 a presents surface received from initial values of function $\varphi(x,y,z)$. Fig. 1 б shows surface received in the result of using the modified interpolar Besselian formula for the function, which depends on three variable function.

Conclusions

Paper presents method for mathematical simulation of three dimensional surfaces with the help of modified interpolar function of three variable by Besselian method.

Suggested mathematical model is pretty simple and efficient, it may be used in different spheres, such as medicine, space researches, geophysics for simulation three dimensional surfaces, interpolation and restoring functions, which describe value, depending on space coordinates.

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Bogach Ilona – Cand.Sc.(Eng) Assist.Prof, Department of automation and information measuring equipment.

Mashnitskiy Maksym – Post graduate, Department of automation and information measuring equipment.

Khomchuk Anatoliy – Senior Lecturer, Department of automation and information measuring equipment.

Vinnytsia National Technical University.