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## **MEASUREMENT OF INTEGRAL PARAMETERS OF NON – LINEAR ELECTRICAL ENGINEERING OBJECTS**

*Conductivities of non linear distortion are introduced, which integrally characterize non linearity of elements, physical fundamentals of their measurement are considered.*

**Key words:** non linear objects , harmonic spectra, parameter measurements.

### **Problem set up**

In non – linear electrical engineering objects, processes of energy consumption are characterized by complete  $S$  , active  $P$  , reactive  $Q$  of powers and distortion power  $D$  ,

$$S = \sqrt{P^2 + Q^2 + D^2} . \quad (1)$$

Distortion power can be written in the form of the sum of orthogonal components:

$$D = \sqrt{D_P^2 + D_Q^2} , \quad (2)$$

which can be expressed by means of static parameters of the elements.

Let us consider non – linear  $RL$  – two – terminal network with series connection of elements, non – linear characteristics of elements are approximated by even power polynomial of the current

$$R(i) = a_0 + a_2 i^2 + \dots ; \quad L(i) = b_0 + b_2 i^2 + \dots ,$$

where  $R(i)$ ,  $L(i)$  – are static resistance and inductance of non – linear resistive and inductive elements correspondingly;  $a_0, a_2, \dots, b_0, b_2, \dots$  – are static parameters of resistive and inductive elements correspondingly (coefficients of power polynomials).

Available non – linear resistive element results in the advent of power component of non- linear distortion  $D_P$ , and available non – linear inductive element leads to the advent of power component of non – linear distortion  $D_Q$ , which can be defined by formulas:

$$D_P = I_1^2 \sqrt{\sum_{k=2}^{\infty} (R_{1k}(I_1))^2} ; \quad D_Q = I_1^2 \sqrt{\sum_{k=2}^{\infty} (X_{1k}(I_1))^2} , \quad (3)$$

where –  $R_{1k}(I_1)$ ,  $X_{1k}(I_1)$  – are resistances, connecting  $k$  – th harmonic of voltage on resistive and inductive elements correspondingly with the first harmonic of current  $I_1$ . The latter express by static parameters [1]:

$$\begin{aligned} R_{11}(I_1) &= a_0 + \frac{3}{4} a_2 I_1^2 + \frac{10}{16} a_4 I_1^4 + \dots ; \\ R_{13}(I_1) &= \frac{1}{4} a_2 I_1^2 + \frac{5}{16} a_4 I_1^4 + \dots ; \quad R_{15}(I_1) = \frac{1}{16} a_4 I_1^4 + \dots ; \quad \dots ; \\ X_{11}(I_1) &= \omega \left( b_0 + \frac{3}{4} b_2 I_1^2 + \frac{10}{16} b_4 I_1^4 + \dots \right) ; \\ X_{13}(I_1) &= 3\omega \left( \frac{1}{4} b_2 I_1^2 + \frac{5}{16} b_4 I_1^4 + \dots \right) ; \quad X_{15}(I_1) = 5\omega \left( \frac{1}{16} b_4 I_1^4 + \dots \right) ; \dots . \end{aligned}$$

In case of parallel connection of non – linear elements power components of non – linear distortion of sinusoidal supply voltage

$$D_P = U_1^2 \sqrt{\sum_{k=2}^{\infty} (g_{1k}(U_1))^2}; \quad D_Q = U_1^2 \sqrt{\sum_{k=2}^{\infty} (b_{1k}(U_1))^2}, \quad (4)$$

where  $g_{1k}(U_1)$ ,  $b_{1k}(U_1)$  – are conductances, connecting  $k$ -th current harmonic on resistive and inductive elements correspondingly, with the first harmonic of voltage  $U_1$ .

But experimental definition of distortion power components  $D_P$ ,  $D_Q$  is possible only applying spectroanalyzers and does not find practical application in systems of automatic control of non – linear electroengineering objects due to low operation speed.

### The aim of research

The aim of the given research is elaboration of the method for definition of distortion power of non – linear electroengineering objects, providing high operation speed

### Substantiation of the results

Simpler, as compared with  $D_P$ ,  $D_Q$  is evaluation of distortion powers correspondingly for serial and parallel connection of non – linear elements using formulas:

$$\hat{D}_P = I_1^2 \sum_{k=2}^{\infty} R_{1k}; \quad \hat{D}_Q = I_1^2 \sum_{k=2}^{\infty} X_{1k}; \quad (5)$$

$$\hat{D}_P = U_1^2 \sum_{k=2}^{\infty} g_{1k}; \quad \hat{D}_Q = U_1^2 \sum_{k=2}^{\infty} b_{1k}. \quad (6)$$

In this case, real values of distortion powers can be defined by their evaluation, taking into account the efficient number of harmonic components from the expressions:

$$D_P^2 = \hat{D}_P^2 / n_P; \quad D_Q^2 = \hat{D}_Q^2 / n_Q, \quad (7)$$

where  $n_P$ ,  $n_Q$  – are efficient numbers of harmonic components, generated by resistive and inductive non – linear elements, correspondingly, which are determined by formulas:

$$n_P = \frac{\left( \sum_{k=2}^{\infty} R_{1k} \right)^2}{\sum_{k=2}^{\infty} R_{1k}^2}; \quad n_Q = \frac{\left( \sum_{k=2}^{\infty} X_{1k} \right)^2}{\sum_{k=2}^{\infty} X_{1k}^2}; \quad n_P = \frac{\left( \sum_{k=2}^{\infty} g_{1k} \right)^2}{\sum_{k=2}^{\infty} g_{1k}^2}; \quad n_Q = \frac{\left( \sum_{k=2}^{\infty} b_{1k} \right)^2}{\sum_{k=2}^{\infty} b_{1k}^2}. \quad (8)$$

Evaluations of non – linear distortion powers are convenient to represent by four components:  $D_{nP}$ ,  $D_{nQ}$ , characterizing the action of odd harmonics, and  $D_{pP}$ ,  $D_{pQ}$ , characterizing the action of even harmonics, with corresponding efficient numbers of harmonic components.

Hence, we obtain the possibility to introduce the components of spectral resistance and spectral conductivity, which characterize non – linearity of the object, in the form of

$$X_{nP} = \frac{D_{nP}}{I_1^2}; \quad X_{nQ} = \frac{D_{nQ}}{I_1^2}; \quad X_{pP} = \frac{D_{pP}}{I_1^2}; \quad X_{pQ} = \frac{D_{pQ}}{I_1^2}; \quad (9)$$

$$b_{nP} = \frac{D_{nP}}{U_1^2}; \quad b_{nQ} = \frac{D_{nQ}}{U_1^2}; \quad b_{pP} = \frac{D_{pP}}{U_1^2}; \quad b_{pQ} = \frac{D_{pQ}}{U_1^2}. \quad (10)$$

Let us consider the possibility of separate determination of spectral resistance components and components of non – linear objects conductivity using current dependence  $i(t)$  and its component,

caused by non – linearity  $i_n(t)=i(t)-i_1(t)$ , where  $i_1(t)$  – is the current of the first harmonic.

Fig 1 shows odd and Fig 2 – even cosine harmonic components of current  $i_n(t)$  in the point  $t=10$  ms ordinates of odd and even cosine harmonic components are added, in point  $t=20$  ms they are subtracted. Thus, we can provide definition and separation of these components if signs of all components are identical (positive or negative). Subtracting instantaneous values of current in these points and dividing into two  $[i_n(t=10\text{ ms})-i_n(t=20\text{ ms})]/2$ , we can provide definition of odd cosine harmonic components. And vice versa, adding instantaneous values of current in these points and dividing into two  $[i_n(t=10\text{ ms})+i_n(t=20\text{ ms})]/2$ , we can provide definition of even cosine harmonic components.

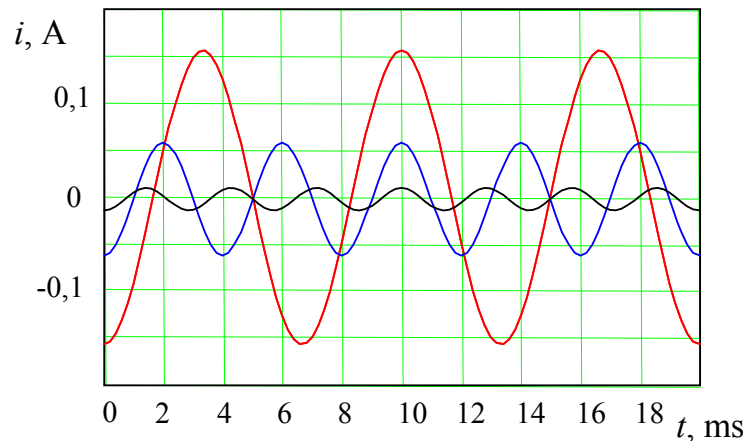


Fig. 1. Odd cosine components of current

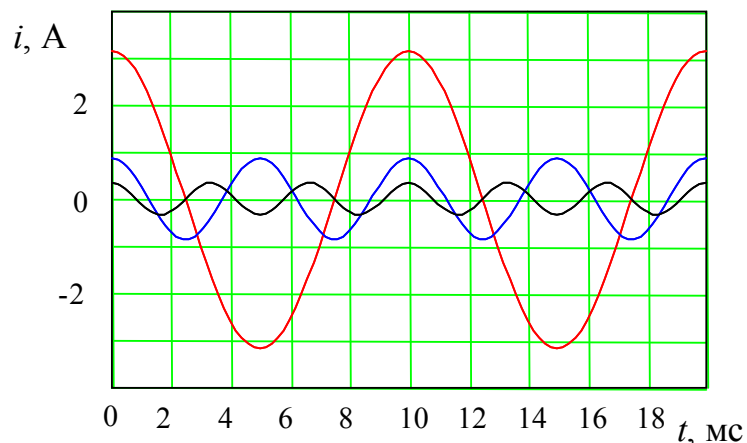


Fig. 2. Even cosine components of current

Note, that even cosine harmonics are not equal to zero in points  $t=5$  ms и  $t=15$  ms. In case of sign variable cosine components, for instance, if the fourth harmonic has negative sign, then instantaneous value of current in these point is equal to negative value of the sum of their amplitude values.

Fig 3 shows dependences of sign variable of odd, and in Fig 4 – even sinusoidal harmonic components of current  $i_n(t)$ . In points  $t=15$  ms ordinates of sign variable odd sinusoidal harmonic components are added. In these points ordinates of even sinusoidal harmonic component equal zero.

But as it was stated above, in these points even cosine harmonic components influence, and in these points this influence is identical. Subtracting instantaneous value of current and dividing into two

$[i_n(t=5\text{ ms})-i_n(t=15\text{ ms})]/2$ , we can provide definition of sign variable odd sinusoidal harmonic

components. And vice versa, adding instantaneous values of current in these points and dividing into two  $[i_n(t=5\text{ms})+i_n(t=15\text{ms})]/2$ , we can provide definition of sign variable even cosine harmonic components. Sign variable odd cosine components, sign constant odd sinusoidal components as well as even sinusoidal components remain non defined. For definition of integral content of these components it is necessary to use spectral phase shifter, which provides phase – shift of each harmonic by the angle of  $\pi/2$  [2].

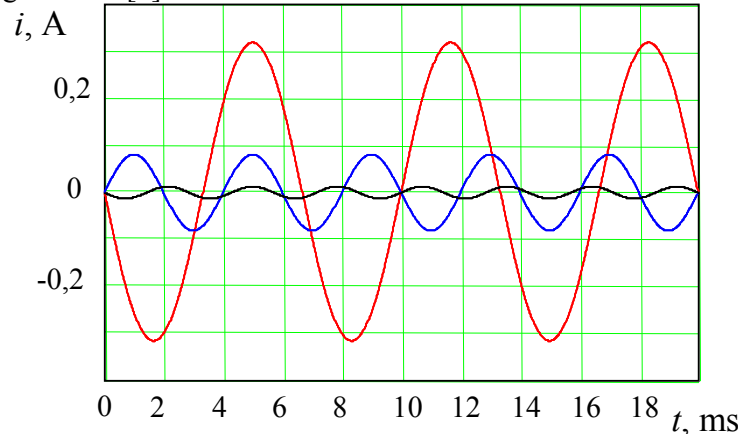


Fig. 3. Odd sign variable sinusoidal components of current

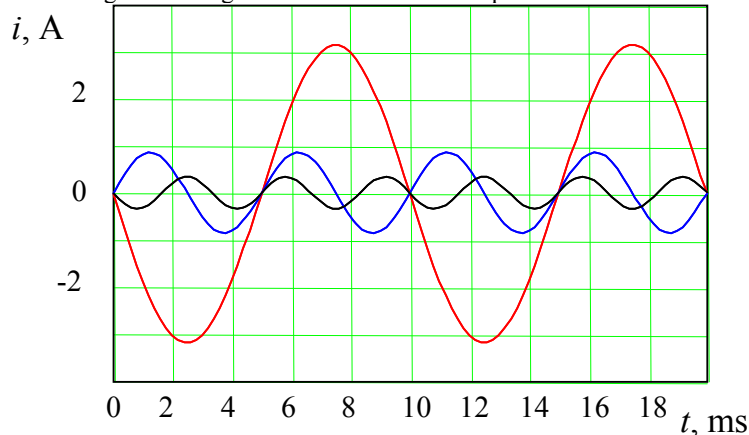


Fig. 4. Even sinusoidal components of current

Fig 5 shows dependences of current  $i(t)$  and its component, caused by non – linearity  $i_n(t)=i(t)-i_1(t)$  of the object, represented by parallel  $rl$  – equivalent circuit with non – linear inductive element, non – linear characteristic of which is shown in Fig 6.

In the given case cosine represented of harmonic spectrum  $u(t)=311\cos\omega t$  is used, harmonic spectrum of the current is shown in Table 1.

Having recorded the value of current component, caused by non- linearity  $i_n(t)$ , in the point  $t=5\text{ ms}$  or  $t=15\text{ ms}$ , we can define integral content of highest odd harmonics  $i_n(t=5\text{ms})=-i_n(t=15\text{ms})=0,414\text{ A}$ . The sum of amplitude values of the third, fifth and seventh harmonics of the current equals 0,412 A (Table 1).

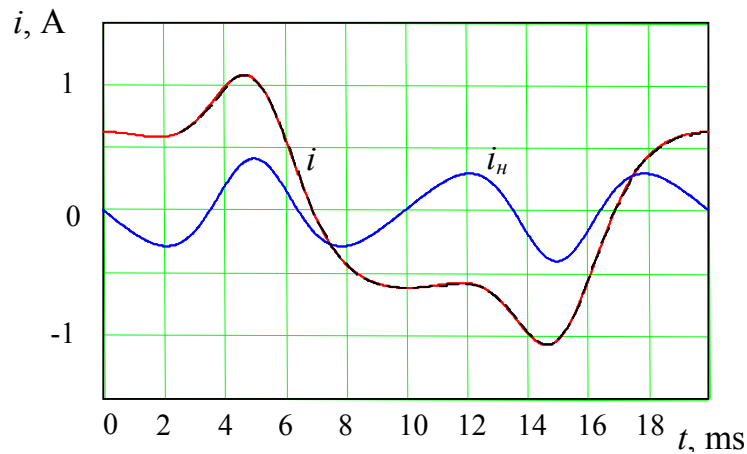


Fig. 5. Dependence of the current across the object with non – linear inductive element.

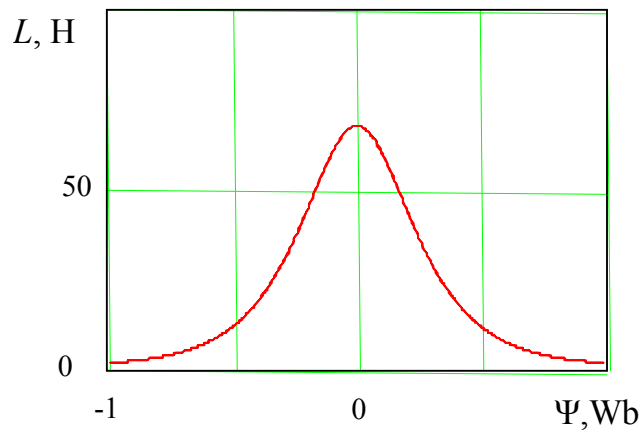


Fig. 6. Non – linear characteristics of inductive element

Table 1

Harmonic spectrum of the current across the object with non – linear inductive element

| $\nu$   | 1     | 2 | 3      | 4 | 5     | 6 | 7      |
|---------|-------|---|--------|---|-------|---|--------|
| $a_\nu$ | 0,620 | 0 | 0      | 0 | 0     | 0 | 0      |
| $b_\nu$ | 0,634 | 0 | -0,318 | 0 | 0,082 | 0 | -0,012 |

Thus, in the given case non – linear objects is characterized by three conductances: active, reactive, and reactive of distortion. Distortion conductance is  $b_{nQ} = \frac{0,414}{220} = 1,88 \cdot 10^{-3} \text{ sm}$ .

Efficient number of harmonic components, generated by inductive non – linear element is  $n_{nQ} = 1,59$ .

Fig 7 shows dependence of current  $i(t)$  and its component, caused by non – linearity  $i_n(t)$  of the object, represented by parallel  $rl$  – equivalent circuit.

Non linear characteristics of resistive and inductive elements are shown in Fig 8, harmonic spectrum of the current is shown in Table 2.

Having recorded the value of current component, caused by non- linearity  $i_n(t)$ , in the point  $t=5 \text{ ms}$  or  $t=15 \text{ ms}$  ( $i_n(t=\text{ms})=-i_n(t=15\text{ms})=0,422 \text{ A}$ ), integral content of highest odd sinusoidal harmonics characterizing non – linearity of inductive element can be defined. The sum of amplitude values of the third, fifth, seventh and ninth harmonics of the current equals 0,421 A (Table 2).

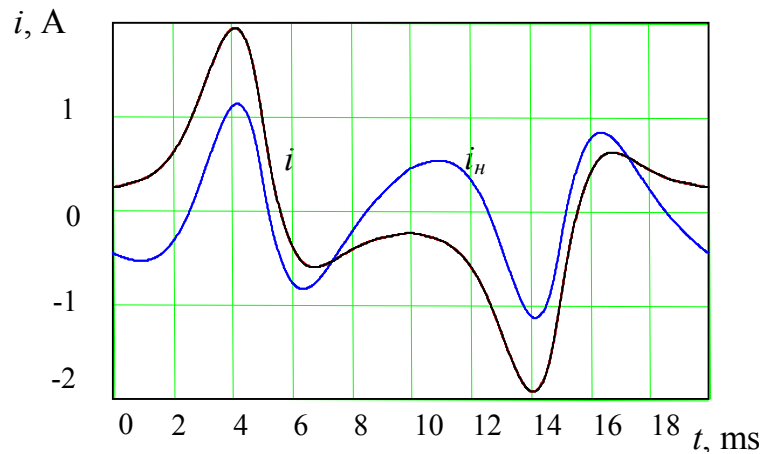


Fig. 7. Dependence of current across the object with non – linear inductive and resistive elements.

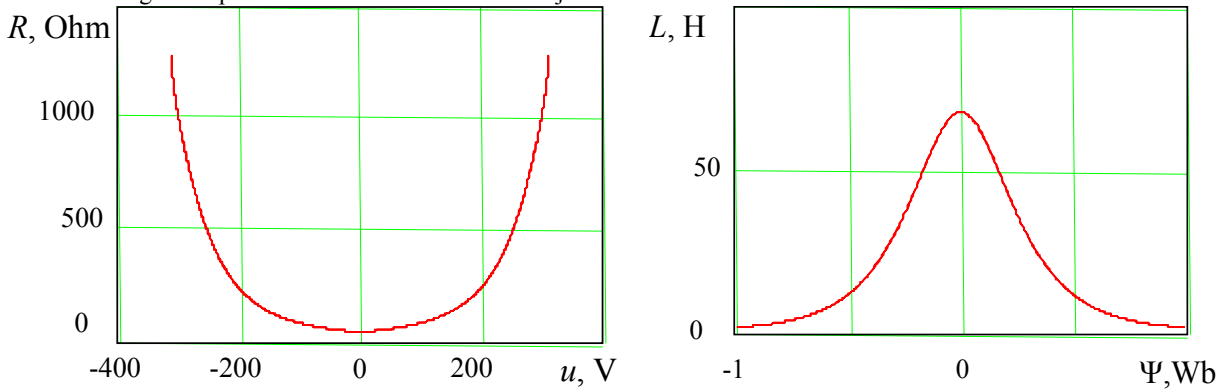


Fig. 8. Non – linear characteristics of resistive and inductive elements

Table 2

**Harmonic spectrum of the current across the object with non – linear resistive and inductive elements**

| $\nu$   | 1     | 2 | 3      | 4 | 5     | 6 | 7      | 8 | 9      |
|---------|-------|---|--------|---|-------|---|--------|---|--------|
| $a_\nu$ | 0,694 | 0 | -0,665 | 0 | 0,309 | 0 | -0,115 | 0 | -0,041 |
| $b_\nu$ | 0,634 | 0 | -0,321 | 0 | 0,085 | 0 | -0,013 | 0 | 0,002  |

To define integral content of highest sign variable odd cosine harmonics, which characterize non – linearity of resistive element, it is necessary to use spectral phase – shifter. Thus, in the given case non – linear object is characterized by four conductances: active and reactive conductances, as well as by two reactive conductances of distortion  $b_{nP} = \frac{1,184}{220} = 5,38 \cdot 10^{-3} \text{ sm}$ ,  $b_{nQ} = \frac{0,422}{220} = 1,92 \cdot 10^{-3} \text{ sm}$ . Effective numbers of harmonic components, which are generated by non – linear elements, will be  $n_{nP} = 2,56$ ,  $n_{nQ} = 1,62$ .

### Conclusions

Non – linear objects, besides active and reactive conductances, can be characterized by four conductance of non - linear distortion: which integrally characterize non – linearity of reactive elements and inductive elements and the character of this non – linearity. Physical fundamentals of their measurement are considered. It is shown that application of instantaneous values of currents allows to measure the parameters of non – linear objects.

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