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METHOD OF IDENTIFICATION OF NON-LINEAR DYNAMIC OBJECTS WITH EXTREME STATIC CHARACTERISTICS

There had been suggested method for identification of non linear dynamic objects with extreme characteristics. For the realization of its algorithm the object must be acted upon initially by the stage and further by sinusoidal input influences, responses of the object must be registered. Calculated relations of the suggested method have been obtained in accordance with the Fourier ideology – integral method identification, the relations assume decomposition of input influence and object response on the influence into sections of Fourier series.

Keywords: *FIMI, Fourier-integral method, identification, nonlinear dynamic objects, extreme static characteristic.*

Initial conditions and problem set up.

Wide class of dynamic objects in chemical technology and heat power engineering with input influence $x(t)$ and influence response $y(t)$, dynamics of which is characterized by pulse transient response $g(t)$, inherent static characteristics $y = f(x)$, which is of extreme character (fig. 1).

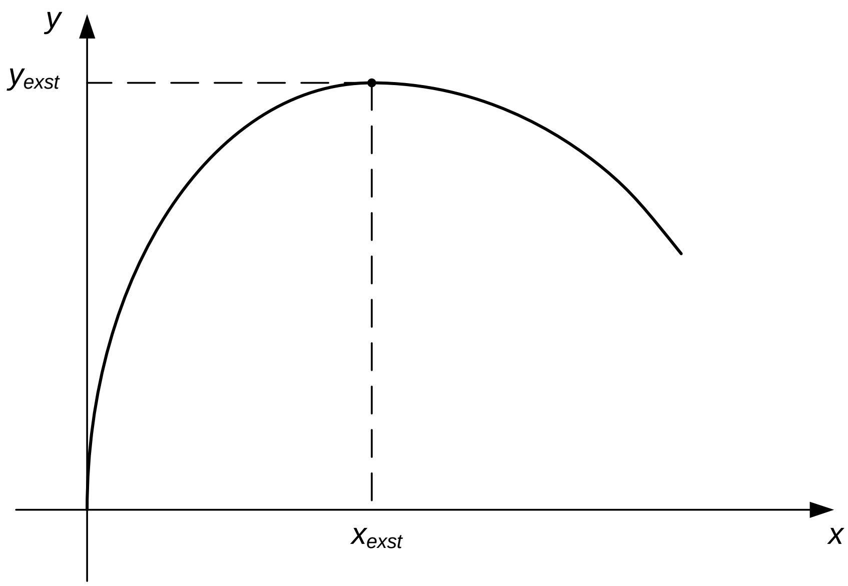

Fig. 1. Extreme static characteristic of dynamic object

Thus for complete identification of such object, it is necessary to determine these characteristics $g(t)$, and $y = f(x)$.

Such problem is considered in the given paper.

For the solution of this problem we will put forward two initial preconditions, in accordance with the first one it is suggested to identify static characteristic $y = f(x)$ of the object by degree polynomial of the third order, i.e.

$$y = c_1 x + c_2 x^2 + c_3 x^3 . \tag{1}$$

Naturally, for approximation of extreme static characteristic, presented in Fig.1, we may use degree polynomial of the second order, since its extreme point will have the same coordinates (x_{exst}, y_{exst}), but we will use degree polynomial of the third order as it describes more accurately the character of coordinate y increase with the initial values of x coordinate.

as the second precondition we will make use of the known approach [1-3], based on the assumption, that the structure of dynamic object is considered in the form of sequential connection of its time lagging linear part with pulse transient response $g(t)$ and the intermediate output signal $x^*(t)$ as well as non-linear lag-free element with $y = f(x^*)$ characteristic (Fig. 2).

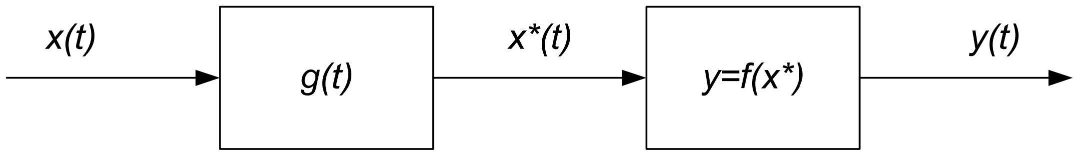

Fig. 2. Structure of dynamic object with allocation of time-lagging linear part and lag-free non-linear part

It is obvious, that after introduction of the second initial condition in equation (1), x instead of x^* should be considered.

Identification of non-linear static characteristic.

As is known [1-3], output signal $x^*(t)$ of linear part of the dynamic object may be found by means of convolution integral

$$x^*(t) = \int_{-\infty}^{\infty} x(t-\tau)g(\tau)d\tau \,, \tag{2}$$

which on condition of physical realization of the object

$$g(t) = \begin{cases} g(t), & t \geq 0, \\ 0, & t < 0, \end{cases} \tag{3}$$

obtains the form

$$x^*(t) = \int_{0}^{\infty} x(t-\tau)g(\tau)d\tau \tag{4}$$

In accordance with Fourier –integral method of dynamic objects identification [3] we decompose the signal $x(t)$ into the section of Fourier series in selected time interval T. According to [3, 4] we obtain

$$x(t) = \sum_{k=-n}^{n} a_k e^{jk\omega_1 t} \,, \tag{5}$$

where $\omega_1 = \frac{2\pi}{T}$, and

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_1 t}dt =$$
$$= \frac{1}{T}\int_0^T x(t)cos(k\omega_1 t)dt - j\frac{1}{T}\int_0^T x(t)sin(k\omega_1 t)dt, \quad k = -n, -(n-1), \ldots, -1, 0, 1, 2, \ldots, n. \tag{6}$$

It should be noted that with the formation of any signal of physical system with limited energy reserve, it must always be approximated with preset accuracy by section of Fourier series.

Substituting values of $x(t)$ from the expression (5) into the expression (4), we obtain

$$x^*(t)=\int_0^{\infty}\sum_{k=-n}^{n}a_k e^{jk\omega_1(t-\tau)}g(\tau)d\tau\,. \tag{7}$$

or

$$x^*(t)=\sum_{k=-n}^{n}a_k e^{jk\omega_1 t}\int_0^{\infty}g(\tau)e^{-jk\omega_1\tau}d\tau\,. \tag{8}$$

If we recollect that transfer function of the linear art of the dynamic object — it is

$$W(p)=\int_0^{\infty}g(\tau)e^{-p\tau}d\tau\,, \tag{9}$$

and its gain-phase frequency characteristic (GPFC) — it is

$$W(j\omega)=W(p)\big|_{p=j\omega}\,, \tag{10}$$

then equation (8) may easily be reduced to

$$x^*(t)=\sum_{k=-n}^{n}a_k W(jk\omega_1)e^{jk\omega_1 t}\,. \tag{11}$$

Substituting values $x^*(t)$ from the expression (11) into expression (1) we obtain

$$\begin{gathered} y(t)=c_1\sum_{k=-n}^{n}a_k W(jk\omega_1)e^{jk\omega_1 t}+\\ +c_2\left(\sum_{k=-n}^{n}a_k W(jk\omega_1)e^{jk\omega_1 t}\right)^2+c_3\left(\sum_{k=-n}^{n}a_k W(jk\omega_1)e^{jk\omega_1 t}\right)^3. \end{gathered} \tag{12}$$

In case if output signal $x(t)$ is sinusoid with frequency ω_1, that is

$$x(t)=A\sin\omega_1 t=A\left(\frac{e^{j\omega_1 t}-e^{-j\omega_1 t}}{2j}\right)=\frac{A}{2j}e^{j\omega_1 t}+\frac{A}{-2j}e^{-j\omega_1 t}\,, \tag{13}$$

then equation (12) will be transferred into equation

$$\begin{gathered} y(t)=c_1\left(a_{-1}W(-j\omega_1)e^{-j\omega_1 t}+a_1W(j\omega_1)e^{j\omega_1 t}\right)+\\ +c_2\left(a_{-1}W(-j\omega_1)e^{-j\omega_1 t}+a_1W(j\omega_1)e^{j\omega_1 t}\right)^2+\\ +c_3\left(a_{-1}W(-j\omega_1)e^{-j\omega_1 t}+a_1W(j\omega_1)e^{j\omega_1 t}\right)^3, \end{gathered} \tag{14}$$

where

$$a_1=\frac{A}{2j}\,,\quad a_{-1}=-\frac{A}{2j}\,. \tag{15}$$

Raising to the power (14) and grouping terms with identical harmonic components, we obtain

$$\begin{gathered} y(t)=2c_2a_{-1}a_1W(j\omega_1)W(-j\omega_1)+\\ +\left(c_1a_{-1}+3c_3a_{-1}^2a_1W(j\omega_1)W(-j\omega_1)\right)W(-j\omega_1)e^{-j\omega_1 t}+\\ +\left(c_1a_1+3c_3a_{-1}a_1^2W(j\omega_1)W(-j\omega_1)\right)W(j\omega_1)e^{j\omega_1 t}+\\ +c_2a_{-1}^2W^2(-j\omega_1)e^{-j2\omega_1 t}+c_2a_1^2W^2(j\omega_1)e^{j2\omega_1 t}+\\ +c_3a_{-1}^3W^3(-j\omega_1)e^{-j3\omega_1 t}+c_3a_1^3W^3(j\omega_1)e^{j3\omega_1 t}. \end{gathered} \tag{16}$$

Now we decompose into sections of Fourier series in the same time interval T output signal $y(t)$, which is the reaction of dynamic object on input sinusoid, that is, we present it in the form

$$y(t)=\sum_{k=-m}^{m} b_k e^{jk\omega_1 t}\ , \tag{17}$$

where $\omega_1=\frac{2\pi}{T}$, and

$$b_k=\frac{1}{T}\int_0^T y(t)e^{-jk\omega_1 t}dt= \\ =\frac{1}{T}\int_0^T y(t)cos(k\omega_1 t)dt - j\frac{1}{T}\int_0^T y(t)sin(k\omega_1 t)dt,\quad k=-3,-2,-1,0,1,2,3. \tag{18}$$

Since in the right-hand side of equation (16) we have only constant component and harmonics with frequencies $-3\omega_1$, $-2\omega_1$, $-\omega_1$, ω_1, $2\omega_1$, $3\omega_1$, then the series (17) for $y(t)$ reaction of the object on the sinusoid of ω_1 frequency will also have only these components — that is why the k values for determination of Fourier coefficient b_k are set only within the limits from–3 to3.

Substituting $y(t)$ value from the expression (17) if $m=3$ into the equation (16), we obtain the identity which will be valid if Fourier coefficients are equal, when harmonics in right-hand side and left-hand side of this identity are the same.

As a result we obtain the following system of equations:

$$\begin{cases} b_0=2c_2 a_{-1}a_1 W(j\omega_1)W(-j\omega_1), \\ b_{-1}=\left(c_1 a_{-1}+3c_3 a_{-1}^2 a_1 W(j\omega_1)W(-j\omega_1)\right)W(-j\omega_1), \\ b_1=\left(c_1 a_1+3c_3 a_{-1}a_1^2 W(j\omega_1)W(-j\omega_1)\right)W(j\omega_1), \\ b_{-2}=c_2 a_{-1}^2 W^2(-j\omega_1), \\ b_2=c_2 a_1^2 W^2(j\omega_1), \\ b_{-3}=c_3 a_{-1}^3 W^3(-j\omega_1), \\ b_3=c_3 a_1^3 W^3(j\omega_1). \end{cases} \tag{19}$$

From the sixth and the seventh equations of this system we find that

$$c_3=\frac{b_{-3}+b_3}{a_{-1}^3 W^3(-j\omega_1)+a_1^3 W^3(j\omega_1)}, \tag{20}$$

from the fourth and fifth equations we obtain

$$c_2=\frac{b_{-2}+b_2}{a_{-1}^2 W^2(-j\omega_1)+a_1^2 W^2(j\omega_1)}, \tag{21}$$

and from the second and third —

$$c_1=\frac{b_{-1}-f_{-1}(a_{-1},a_1,c_3,W)+b_1-f_1(a_{-1},a_1,c_3,W)}{a_{-1}W(-j\omega_1)+a_1 W(j\omega_1)}, \tag{22}$$

where

$$\begin{cases} f_{-1}(a_{-1},a_1,c_3,W)=\left(3c_3 a_{-1}^2 a_1 W(j\omega_1)W(-j\omega_1)\right)W(-j\omega_1), \\ f_1(a_{-1},a_1,c_3,W)=\left(3c_3 a_{-1}a_1^2 W(j\omega_1)W(-j\omega_1)\right)W(j\omega_1). \end{cases} \tag{23}$$

It is obvious that the equation of the system (19) is redundant — it may be used as correctness criterion of problem solution.

One more criterion of the correctness of problem is the appearance of basically different from zero values of Fourier coefficients with numbers $k > 3$ and $-k < -3$ in the output signals $y(t)$ during its decomposition in series (17) on condition that only sinusoid of one frequency ω_1 enters the object input. It means that real static characteristic $y(x)$ of the object must be approximated by degree polynomial of the order higher than the 3d. If harmonics with frequency $j4\omega_1$ and $-j4\omega_1$ in the signal $y(t)$ for approximation of characteristic $y = f(x)$ polynomial must be of the 4-th order, and if harmonics with frequency $j5\omega_1$ and $-j5\omega_1$ are available, this polynomial must be of the fifth order, and further by increase.

It should be noted that the increase of polynomial order for approximation of $y = f(x)$ characteristics does not significantly complicates the receiving of expressions for calculation of factors of this polynomial on condition of using the one frequency sinusoidal on the objects input, since for receiving the correlations of the kind (19) and in this case in the expression of the kind (14) the power is evaluated following the binomial theorem.

From the correlations (20), (21), (22) follows, that it is possible to identify the extreme static characteristics $y = f(x)$ of the dynamic object of the class under consideration on condition that the value of GPFC is known $W(j\omega)$ of the linear inertial part of this object on frequencies ω_1 and $-\omega_1$, that is the known is $W(j\omega_1)$ and $W(-j\omega_1)$. The second part of the work will describe the way to find these values.

Identification of linear inertial part of dynamic object

From the physics of any dynamic object it follows that the processes in this object grow linearly with the exception of the no sensitive zone, close to zero, or backlash until its mass or energetic volume is at least half filled up with mass or energy, coming together with input influence.

In other words, if we form the input influence on the object in the way its level meets the steady-state level of response of this object to the applied input influence (fig. 3), it allows to state that within the range of values of input coordinate $y(t)$ from $0{,}1y_{st-s}$ to $0{,}5y_{st-s}$ the object will behave as the linear one.

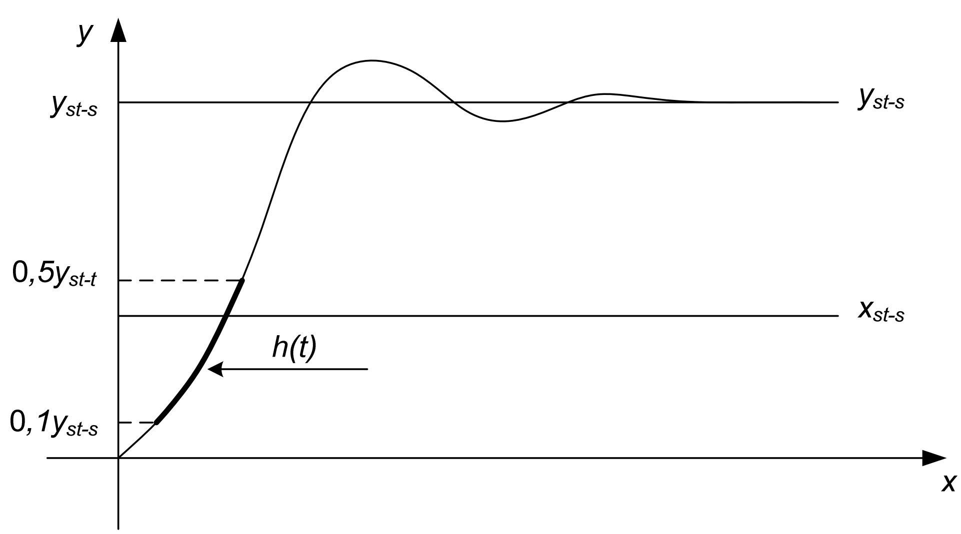

Fig. 3. Diagram of transitional process in the dynamic object after submitting the staged influence on the level of its steady-state value to the input

And if the input influence is submitted by jump from 0 to x_{st-s}, then in the given range of values $y(t)$ the output coordinate will coincide with the transitional characteristics $h(t)$ of the linear frequency of this object, which, for the object with one volume of energy or mass concentration, looks as follows:

$$h(t) = K\left(1 - e^{-\frac{t}{T_1}}\right), \tag{24}$$

for the object with the two volumes of concentration of energy and mass —

$$h(t) = K\left(1 - \lambda_1 e^{-\frac{t}{T_1}} - \lambda_2 e^{-\frac{t}{T_2}}\right), \quad \lambda_1 + \lambda_2 = 1, \tag{25}$$

or

$$h(t) = K\left(1 - e^{-\frac{t}{T_1}} \cos\beta\, t\right), \tag{26}$$

and for the object with three volumes of concentration of energy and mass —

$$h(t) = K\left(1 - \lambda_1 e^{-\frac{t}{T_1}} - \lambda_2 e^{-\frac{t}{T_2}} - \lambda_3 e^{-\frac{t}{T_3}}\right), \quad \lambda_1 + \lambda_2 + \lambda_3 = 1, \tag{27}$$

or

$$h(t) = K\left(1 - \lambda_1 e^{-\frac{t}{T_1}} - \lambda_2 e^{-\frac{t}{T_2}} \cos\beta\, t\right), \quad \lambda_1 + \lambda_2 = 1. \tag{28}$$

For the majority of volumes of concentration of energy or mass, the range of approximated functions for $h(t)$ may by continued using the same principle. However there is no necessity in this since in the tasks of synthesis of systems of extreme regulation it is not expedient for the transient function of the linear part of dynamic object to be over the third order, as the regulation system does not allow for a long distance between the working point and the extreme point.

It is obvious, that for all the listed above expressions for $h(t)$ parameter K may be found from the expression

$$K = \frac{y_{st-s}}{x_{st-s}}. \tag{29}$$

So for the approximation of $h(t)$ in the kind of (24) it is necessary from the curve $y(t)$ (see fig. 3) using only one point in the range $0{,}1 y_{st-s} \le y(t) \le 0{,}5 y_{st-s}$ for the determination of the parameter T_1. Obviously, it may be found form the equation

$$h(t_1) = K\left(1 - e^{-\frac{t_1}{T_1}}\right). \tag{30}$$

For the approximation $h(t)$ in the kind of (26) for the parameters T_1 and β it is necessary to use two points from this range $[0{,}1 y_{st-s}; 0{,}5 y_{st-s}]$. For them the system of the equations will look as

$$\begin{cases} h(t_1) = K\left(1 - e^{-\frac{t_1}{T_1}} \cos\beta\, t_1\right), \\ h(t_2) = K\left(1 - e^{-\frac{t_2}{T_1}} \cos\beta\, t_2\right). \end{cases} \tag{31}$$

The same way allows to calculate the parameters of all the other approximations of the characteristic $h(t)$.

After that, using some (for example, M) points of the range $[0{,}1y_{st-s};0{,}5y_{st-s}]$ with the index l, it is necessary to calculate for all the found approximations $h_i(t)$, $i=\overline{1,r}$ the mean square Δ_i of approximation error following the expression

$$\Delta_i = \sum_{l=1}^{M} \left(y_l^* - h(t_l)\right)^2, \quad i = \overline{1,r}, \tag{32}$$

where y_l^* — value of the transient characteristic, taken from the experimental diagram (see. fig. 3), and $h(t_l)$ — value of this characteristics, calculated according to the approximated expression.

The number $\Delta_1, \Delta_2, \ldots, \Delta_r$, which appears to be the least, will determine the best approximation of the transient characteristics $h(t)$ of the linear part of the dynamic object according to the minimum criterion of errors square sum on «fresh» points.

Knowing the transient characteristic $h(t)$ of the linear part of the object allows to find its transient function easily $W(p)$, as[1]

$$g(t) = \frac{dh(t)}{dt}, \tag{33}$$

and for receiving $W(p)$ as for the known $g(t)$ it is necessary to use the expression (9).

In its turn, the expression $W(j\omega_1)$ and $W(-j\omega_1)$, which we need to identify the extreme static characteristic $y = f(x)$, we receive by the direct substitution in the expression for the transient function $W(p)$, received by transformation (9), instead of operator p values $j\omega_1$ and $-j\omega_1$.

Certainly, if after some same type experiments of supplying to the input of the object the equal level input influence of the staged character, we will receive the experimental curves $y(t)$, which will not coincide in the range $[0{,}1y_{st-s};0{,}5y_{st-s}]$, then the parameters of approximations $h(t)$ in the kind of (24) – (28) are to be calculated, using the standard procedure of the method of least squares [3].

Conclusions

1. There had been suggested the method for identification of non-linear dynamic objects with the extreme characteristics, the algorithm of which stipulates for feeding the stepped signal and then the one frequency sinusoid on the objects input.

2. Following the given method, the linear inertial part of the dynamic object shall be identified by the transitional characteristics $h(t)$ with the further transition to the impulse transitional characteristics $g(t)$ and transient function $W(p)$ and GPFC $W(j\omega)$, and the non-linear non-inertial statistic characteristics $y = f(x)$ of the object are identified by the power polynomial.

3. During the calculation of calculation correlations of the suggested method there had been used the ideology of Fourier – integral method of identification (FIMI), developed in the 80-th years of

the previous century by Borys I. Mokin.

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