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MATHEMATICAL MODELING OF LIGHT TRANSMISSION CHARACTERISTICS BY MULTILAYER PERIODIC STRUCTURES IN COMPLEX SYSTEMS

Mathematical modeling is performed, spectral characteristics of light transmission by multilayer periodic structures are obtained. It allowed to select material and geometry of semiconductor structure for optic modulator.

Key words: *modeling, light transmission characteristics, optic modulator, method of transition probabilities, multilayer structure, photonic gap band.*

As a result of rapid development of information technologies, a number of problems, connected with transfer of large volumes of information and computational facilities intended of information processing emerge.

The problem dealing with development of new approaches to realization of computational systems elements and information networks which world enable to increase data transmission rate and processing also emerges.

Existing telecommunication systems to great extent depend on modulators, coding data in light beams. Modulators rapidly change their refection ability, regulating intensity of beams, passing across them.

But modern modulators have a drawback: they cannot operate faster than electronic circuits, which control these modulators. To increase the rate of data transmission, alternative control technologies are required. The given paper considers the possibility of optically-controlled transparent for laser radiation of operations in computational devices.

Quite recently development of photonic bandgaps in periodic structures, but today this direction of development evokes great interest among scientists, paving the way to qualitative breakthrough in technologies, due to transition to all-optic methods of data transfer and processing. Such structures, being in a way, “optic analogues” of semiconductors. characteristic feature of them being the presence of electronic bandgaps, became known as photonic crystals.

Combination of materials with photonic gap bands and non-linear optics is of great importance for creation of optical space-time modulator (controlled transparant). Photonic bandgaps formed in periodic structure provide the possibility of efficient control of light flux.

Crystals can be tuned due to dependence of optic properties of nonlinear materials, which can be used as composites of photonic crystals, on the intensity of radiation. It enables to control light flux, localized in photonic crystal, and create new active elements (switches, modulators), which can be integrated in multifunctional photonic integrated circuits. Switching of properties in photonic crystal can be performed, for instance, by means of external fields (electric or magnetic), as well as a result of using temperature dependence of reflection factor or electro optic effect.

Nanotechnological fabrication of photonic crystals allows to obtain, at the expense of layer thickness of tens nanometers of order at low voltages, intensity of electric field (of the order 10^5 V/m), required for nonlinear effects. Besides, materials with photonic bandgaps can be used for considerable improvement of nonlinear characteristic of the structure due to strict localization of radiation. It makes available nonlinear effects for commercial small-power semiconductor lasers.

Naturally, major part of research is devoted to 1 D photonic crystals. They can be rather easily manufactured, accurately controlling the thickness of layers and even surface, besides, such structures are economically efficient and more simple for design and manufacturing [1].

Nowadays the process of fabrication of 3D photonic crystals with required structure is rather complicated, since it is very difficult to control the value of dielectric permeability with space exactness, compared with the light wavelength. However, due to constantly increasing requirements to laser in various spheres of application, laser beam is close to flat waves. In this case there is no need in 3D photonic crystals, since we can use 1D crystal, that is, multilayer structures.

Hence, we will consider 1D periodic $AlGaAs/GaAs$ and $AlGaAs/AlAs$ structures, which are periodically repeated pairs of layer of given semiconductors. For calculation of photonic bandgap of such structures the method of transient matrices is applied

Method of transient matrix

If we consider only linear processes, for definition of complex transition factor of 1D structure, well – known method of transient matrix is applied. The transition matrix is composed to connect complex amplitudes of right side waves of the structure with complex amplitudes of the same waves of left side. The given method is used while coherent interaction of waves, reflected from each boundary of media division, in order to define the established field [1].

Methods of transient matrices are convenient for geometries, where elements are of infinite length and limited, thickness [2]. If layers thickness is 5 % of periodic structure side length x and y directions (periodicity is only in z direction), the length of the structure in these directions can be considered as infinite. In this case, there is no need to consider boundary conditions on transitions in directions of above – mentioned axes.

For creation of transition matrix two types of matrices are used: gap matrix for transition across the boundary of media division and extention matrix for transition through the layer to the next division boundary.

It was assumed that reflective index varies only z direction, which is the direction of propagation. Reflective index is assumed to be constant I directions x and y. Let us denote by E_m^+

2D vector, consisting of complex amplitude and waves phase, propagating in right – side and left – side directions, on the right of division boundary of *m*- th layer (fig. 1):

$$E_m^+ = \begin{pmatrix} E_m^{+R} \\ E_m^{+L} \end{pmatrix}, \quad E_m^- = \begin{pmatrix} E_m^{-R} \\ E_m^{-L} \end{pmatrix}. \tag{1}$$

Thus, incident wave E_I, reflected wave E_R and transition wave E_T, will have the following form:

$$\begin{aligned} E_I &= E_i e^{i(kz-\omega t)} = E_0^{-R} e^{i(kz-\omega t)} \\ E_R &= E_r e^{i(kz+\omega t)} = E_0^{-L} e^{i(kz+\omega t)} \\ E_T &= E_t e^{i(kz-\omega t)} = E_4^{+R} e^{i(kz-\omega t)}. \end{aligned} \tag{2}$$

1.1. Gap matrix

First we will consider the gap matrix. This matrix must satisfy there different boundary conditions for division boundary between m th and $(m+1)$ th layers. The first conditions is that complete electric field must be continuous across this boundary:

$$E_m^{-R} + E_m^{-L} = E_{m+1}^{+R} + E_{m+1}^{+L}. \tag{3}$$

The following boundary conditions is that the wave is propagating in right – side direction, from the right side of division boundary must be equal to the sum of wave, falling on the boundary in right – side direction from the left side of this boundary, and wave, reflected from this boundary, which is propagating in left- side direction:

$$E_{m+1}^{+R} = r_{m+1,m} E_{m+1}^{+L} + t_{m,m+1} E_m^{-R}. \tag{4}$$

Here $r_{m,k}$ and $t_{m,k}$ – are Frenel coefficients at normal falling for reflection and transition from the layer m into the layer k :

$$r_{m,k} = \frac{(n_m - n_k)}{(n_m + n_k)}, \quad t_{m,k} = \frac{2n_m}{(n_m + n_k)}. \tag{5}$$

Similarly to the second condition, the third boundary condition is that the wave, propagating in left – side direction from the left side of division boundary must be equal to the sum of missing wave, which propagates in left – side direction from the right side of division boundary and reflected from this boundary wave, which propagates in right – side direction.

$$E_m^{-L} = r_{m,m+1} E_m^{-R} + t_{m,m+1} E_{m+1}^{+L}. \tag{6}$$

Fig 1 shows the examples of the structure mode up of three layers, having different refractive index, which illustrates waves distribution and boundary conditions while transition from one layer into another.

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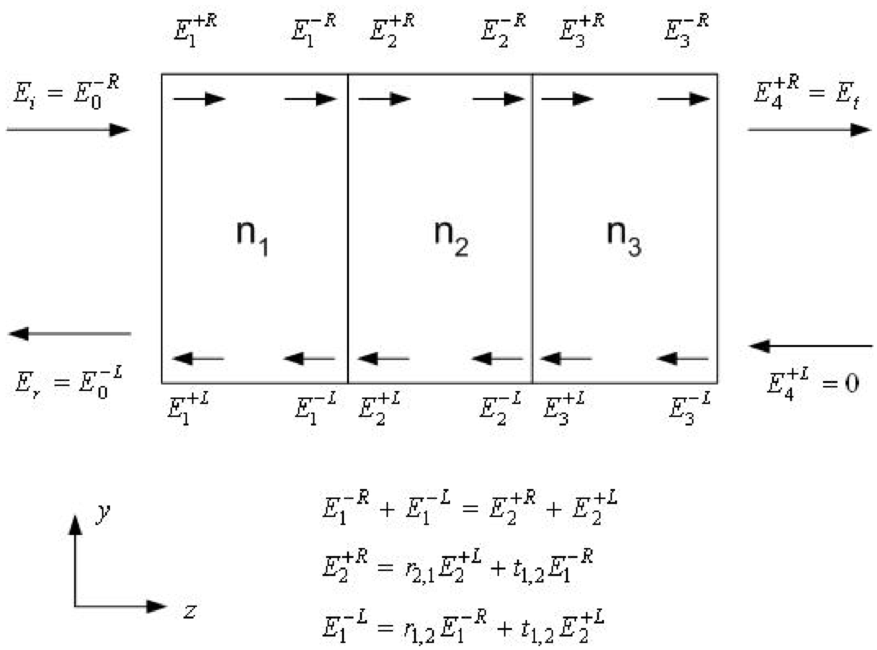

Fig. 1. Illustration of transition matrix method

Applying simple algebraic transformations, we can unite these three conditions in matrix form:

$$\begin{pmatrix} E_{m+1}^{+R} \\ E_{m+1}^{+L} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + n_{m+1}/n_m & 1 - n_{m+1}/n_m \\ 1 - n_{m+1}/n_m & 1 + n_{m+1}/n_m \end{pmatrix} \begin{pmatrix} E_m^{-R} \\ E_{m+1}^{-L} \end{pmatrix}. \tag{7}$$

Hence, discontinuity matrix will have the following form:

$$\Delta(m, m+1) = \frac{1}{2} \begin{pmatrix} 1 + n_{m+1}/n_m & 1 - n_{m+1}/n_m \\ 1 - n_{m+1}/n_m & 1 + n_{m+1}/n_m \end{pmatrix}, \tag{8}$$

then $E_{m+1}^{+} = \Delta(m, m+1) E_m^{-}$ [3].

1.2. Propagation matrix

Now we will consider propagation matrix. This matrix must take into consideration the phase, that changes passing across each layer. Rations between fields at each side of *m*- th layer have the following form:

$$E_m^{-R} = E_m^{+R} e^{i\frac{2\pi n_m}{\lambda} d_m},$$
$$E_m^{-L} = E_m^{+L} e^{-i\frac{2\pi n_m}{\lambda} d_m}. \tag{9}$$

in equation (9) d_m – is physical thickness of m-th layer, λ –is light length in vacuum.

Transforming this into matrix form, we obtain propagation matrix:

$$\Pi(m) = \begin{pmatrix} e^{i\frac{2\pi n_m d_m}{\lambda}} & 0 \\ 0 & e^{-i\frac{2\pi n_m d_m}{\lambda}} \end{pmatrix}, \tag{10}$$

then $E_m^- = \Pi(m) E_m^+$.

Combination of propagation matrix and discontinuity matrix can be used for description of any discrete profile of restrictive factor, applying them for computation of field propagation within the structure. As the example of restrictive profile, shown in Fig, the field from the left side of the last division boundary in the given structure, can be expressed by means of incident field with the following sequence of steps:

$$\begin{aligned} E_1^+ &= \Delta(0,1) E_0^- \\ E_1^- &= \Pi(1) E_1^+ \\ E_2^+ &= \Delta(1,2) E_1^- \\ E_2^- &= \Pi(2) E_2^+. \end{aligned} \tag{11}$$

Applying the same procedure we obtain the final result:
$E_4^+ = \Delta(1,0)\Pi(3)\Delta(2,3)\Pi(2)\Delta(1,2)\Pi(1)\Delta(0,1)E_0^-$ [3].

If we denote the transition matrix for the whole structure as $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$, we obtain:

$$M = \Delta(1,0)\Pi(3)\Delta(2,3)\Pi(2)\Delta(1,2)\Pi(1)\Delta(0,1),$$

$$E_4^+ = M E_0^-.$$

Accumulation of the product in transition matrix does not depend on the direction of motion in the structure, but requires the account of layers in the sequence, they are located. In the given case, matrix is formed by serial motion from the last division boundary of media to the very first one [4].

Since $E_4^+ = \begin{pmatrix} E_4^{+R} \\ E_4^{+L} \end{pmatrix} = \begin{pmatrix} E_t \\ 0 \end{pmatrix}$ and $E_0^- = \begin{pmatrix} E_0^{-R} \\ E_0^{-L} \end{pmatrix} = \begin{pmatrix} E_i \\ E_r \end{pmatrix}$, it may be written:
$E_0^- = \begin{pmatrix} E_0^{-R} \\ E_0^{-L} \end{pmatrix} = \begin{pmatrix} E_i \\ E_r \end{pmatrix}$, i.e.

$$\begin{cases} E_t = m_{11} E_i + m_{12} E_r \\ 0 = m_{21} E_i + m_{22} E_r. \end{cases}$$

$$E_r = -\frac{m_{21}}{m_{22}} E_i, \tag{12}$$

$$E_t = m_{11} E_i + m_{12} \cdot \left(-\frac{m_{21}}{m_{22}}\right) E_i = \left(m_{11} - \frac{m_{12} m_{21}}{m_{22}}\right) E_i. \tag{13}$$

The expression obtained connects the field at the output of the structure with falling input field, taking into account Frenel's reflection and phase, accumulated during passage. Relation of voltages modulii square of the field and falling field will give the transition factors of the structure, which, in its turn, will show wavelength ranges within which light does not propagate in the structure – photonic bandgaps [5].

2. Results

Let us consider the structure, composed of $Al_{0.9}Ga_{0.1}As$ and $GaAs$ layers, periodically repeated.

For gap band orientation on the wavelength $\lambda = 1500$ nm, we selected layers thickness, applying Bragg condition [5]:

$$d \cdot n = \frac{\lambda}{4}, \tag{14}$$

where d – is layer thickness, n – is refractive index of the layer.

That is, such structure consists of so-called half- wave plates and is distributed Bragg mirror.

For $n(Al_{0.9}Ga_{0.1}As) = 3{,}008$ and $n(GaAs) = 3{,}535$, we obtain thicknesses of $d_1 = 124{,}7$ nm and $d_2 = 106{,}1$ nm.

By means of Math Cad package graphical representation of gap band was obtained (Fig 2).

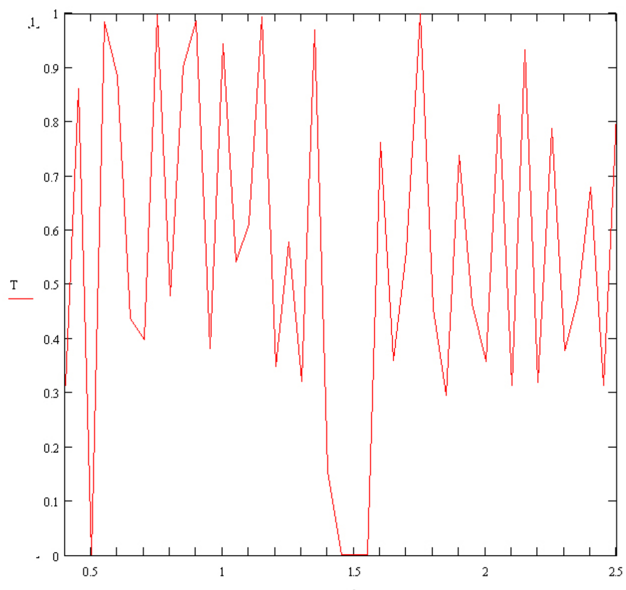

Fig. 2. Dependence of permeability factors of 30 paired $Al_{0.9}Ga_{0.1}As/GaAs$ layers structure on incident radiation wavelength

Photonic gap band is located near 1,5 mkm wavelength. Fig. 3 and Fig. 4 shows gap bands for $Al_{0.2}Ga_{0.8}As/Al_{0.9}Ga_{0.1}As$ ($n = 3{,}467/3{,}04$) and $Al_{0.38}Ga_{0.62}As/AlAs$ ($n = 3{,}345/2{,}968$). Step of the grid on the abscissa axis is 50 nm.

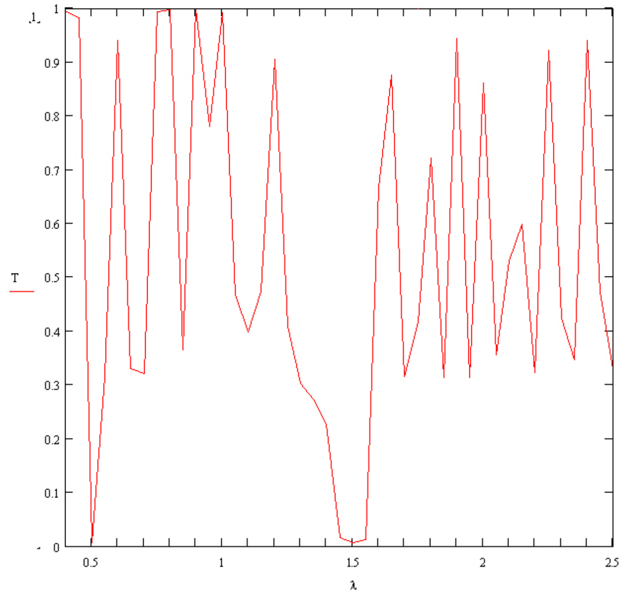

Fig. 3. Dependence of permeability factor of 24 - paired $Al_{0.2}Ga_{0.8}As/Al_{0.9}Ga_{0.1}As$ layers structure on incident radiation wavelength.

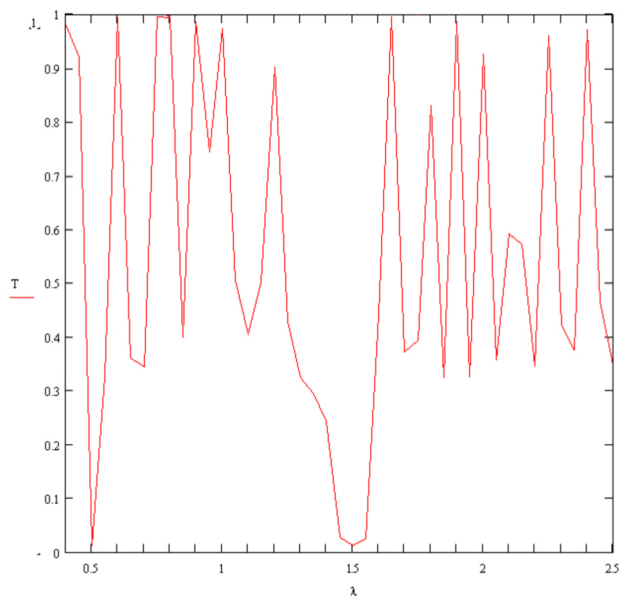

Fig. 4. Dependence of permeability factor of 24- paired $Al_{0.38}Ga_{0.62}As/AlAs$ layers structure on incident radiation wavelength

3. Conclusion

Developed technology of thin films growing of investigated materials enables to select randomly the number for pairs of layer, proceeding from the last required permeability factor inside gap band. Width of gap band depends mainly on the contrast of refractiver factors variations. Among considered materials the structure based on $Al_{0.9}Ga_{0.1}As/GaAs$ has the largest width of gap band. Having reduced discrete counts on wavelength, we obtained the range, where permeability factor for this material does not exceed 0,1: $1{,}43 \le \lambda \le 1{,}58$. Wavelengths range, in which modulator can operate on the base of such structure, is $\Delta\lambda = 150$ nm. For wave multiplexing with $\delta\lambda = 3$ nm operation of the device with 50 channels is possible.

Investigated mathematical model can efficiently be used for obtaining spectral characteristics not only of periodic structures but multilayer structures of random geometry with – faulty layers, which are very attractive for creation of optic filters.

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