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ANALYSIS OF METHODS FOR NORMALIZATION OF VECTORS OF NORMALS FOR THE TASKS OF FORMATION OF THREE DIMENSIONAL IMAGES

There had been made the analysis of the effective methods normalization of vectors of normals for the tasks of computer graphics . There had been given the main methods for determination of vectors of normals for surfaces which are set analytically in the kind of data poligon.

Key words: *vectors of normal, normalization of vectors of normals, spherical angular interpolation.*

Introduction

For achievement of photorealism in the computer graphic it is necessary to renew the properties of surface and to describe the effects of illuminations on the scene from physical point of view. Determination of normal vectors is one of the basic operations, which enables to execute it. The calculation of normal vectors is a component of majority methods of formation of rough and relief surfaces. In the method of bamp – mapping [1], for instance, normal vectors are calculated for determination of surface illumination per pixel. Degree of the illumination of dot depends on an angle between the normal line and beam of light: the smaller the angle, the greater the illumination of this point of the surface.

For rough surface the normals in each point will be different. Namely this principle is a base of a bamp – mapping method. And the method of normal – mapping bases on using of maps of normals, where all the normal vectors are standardized. Color of image shows how the normal of surface is oriented on that pixel of image. In a fact, the normal map sets the surface geometry by virtual rotation of normal's direction in the given pixel. Unlike the maps of roughness (bamp - maps) the effect from maps of normals is seen quite well even in a very glossy material. Regarding the methods of formation of rough surfaces, which use the maps of displacement [2], they also require the calculation of normals, since they set the displacement of surface's dots lengthwise the normal.

Normal vectors set the positions of the observer and the source of light. The vector of normal to the surface sets their local curvature. While painting, applying Fong's method, along the line, being scanned, normals vector values are interpolated, then these normals are further used in Fong's illumination model (which still remains very popular and is widely used in computer graphic) for computation of diffusive and speculative component of surface point illumination.

This helps achieve the better local approximation of the curvature of the surface and correspondingly, more real image.

According to the formula of covering over, the normalization of vectors of normals is needed.

Normalization of vectors of normals [3] requires the execution of the three operations of division three operations of multiplication, two operations of addition and operation of square root finding. It allows to state that the vector operations take a significant part in the calculation process. Thus, the simplification of the procedure of normalization with the aim of its hardware realization is a very important.

Objective of the research

The objective of the given research is the analysis of methods for normalization of normal vectors for using in the tasks of computer graphics.

Vector of normal. Finding vector of normal

Vector of normal, or normal to the flat – is vector, which is perpendicular to this flat . The normal to the non flat surface in some point P is vector, perpendicular to the tangent plane to this surface in point P (see fig. 1).The direction of normal determines the orientation of surface in the space. In computer graphic the normal vector is used for the imitation of geometrical detail on flat surfaces. In this case, the function will determine the smallest deviation from the real direction of normal in each point of surface with an aim of creation of gleams and shaded sections.

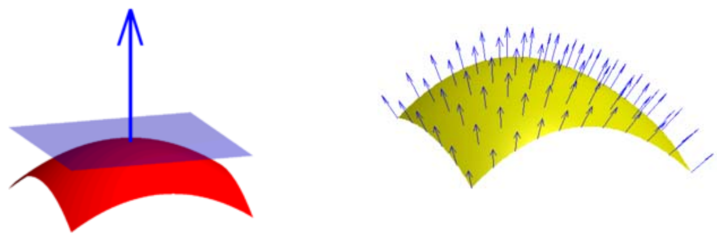


Fig. 1. Determination of vector of normal for the non flat surfaces

On the ideal sphere, for example, the normal to the point on the surface has the same direction, as the vector from the center of the sphere to this point. For the other types of surfaces there are other better ways for finding normal, which depend on how the surface is set.
It is necessary to note that the smooth surface are approximated with bigger number of small flat of target grounds. If vectors, perpendicular to these target grounds are used as normal of approximated surfaces, than the surfaces themselves look segmented, since the space of vectors is not the continuous beyond the bounds of target grounds. But in many cases for the model there is the exact mathematical description, and it is possible to calculate the vector of veritable normal in each point. The use of veritable normal significantly improves the visualization results (fig. 2).

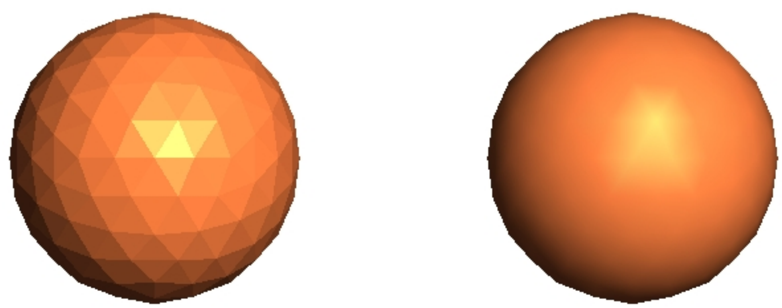


Fig. 2. Veritable normal (to the right) against polygonal normal (to the left)

Calculation of normal for analytical surfaces

Analytical surfaces – are smooth surfaces, which are described by mathematical equation (or some set of equations). In many cases the normal is easy to find for analytical surfaces, for which there is the exhausting description in such a form:

$$V(s, t) = [X(s, t)\ Y(s, t)\ Z(s, t)],$$

where s and t determined in some space, and X, Y and Z – differentiated functions of two variables. To find a normal, it is necessary to calculate the partial derivatives $\frac{\partial V}{\partial s}$ and $\frac{\partial V}{\partial t}$, which are vectors, tangent to the surface in the direction s and t. Their vector derivative product $\frac{\partial V}{\partial s} \times \frac{\partial V}{\partial t}$ perpendicular to both of them and as a result, perpendicular to surface. The following formula reflects the process of calculation of vector product of two vectors:

$$[v_x v_y v_z] \times [w_x w_y w_z] = [(v_y w_z - w_y v_z)(v_z w_x - w_z v_x)(v_x w_y - w_x v_y)]. \tag{1}$$

For the normalization of resulting vector [x, y, z] it is necessary to find its length:

$$l = \sqrt{x^2 + y^2 + z^2} \tag{2}$$

and divide each of the vector components by it.

As an example for such calculations we take the analytical surface $V(s,t) = [2s \quad t^2 \quad 4 + 2st]$.

We calculate partial derivative and with the help of the formula (1) we find vector product of the received vectors:

$$\frac{\partial V}{\partial s} = [2 \quad 0 \quad 2t], \qquad \frac{\partial V}{\partial t} = [0 \quad 2t \quad 2s], \qquad \frac{\partial V}{\partial s} \times \frac{\partial V}{\partial t} = [-4t^2 \quad -4s \quad 4t].$$

Thus if in the given example s=2 и t=1, the correspondent point of the surface is the point with coordinates (4, 1, 8), and vector (-4, -8, 4) will be the vector of normal.

The length of this vector equals 9.8, then the vector of normal of single length equals (-4/9.8, -8/9.8, 4/9.8)=(-0.408163, -0.816326, 0.408163).

The calculation of normal on polygonal data

The surfaces are very often set in the kind of polygonal data. For them to look smooth but not segmented, it is necessary to calculate normal to the surface. In the majority of cases the easiest way to do it is to calculate the vector of normal for each polygonal fragment of the surface with further averaging normals of neighboring fragments. Fig 3 presents the surface and its polygonal approximation.

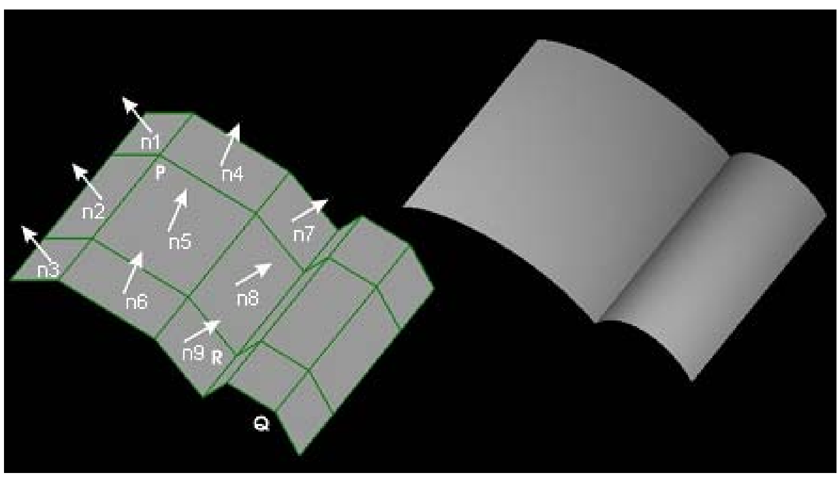


Fig. 3. averaging of normal vectors

To find the vector of normal for the flat polygon we need three any tops of polygon v_1, v_2 and v_3, which do not lie on one straight line. Vector product

$$[v_1 - v_2] \times [v_2 - v_3] \tag{3}$$

will be the normal to the polygon. The resulting vector, as a rule, needs to be normalized..

Than it is necessary to average the normals of neighboring polygons in order not to allow more weight to one of them. For example, if in case, shown on fig. 3, n_1, n_2, n_4 and n_5 – normals of polygons, which unite in the point P, then it is necessary to find the vectors sum $n_1+n_2+n_4+n_5$, and then normalize the received vector. The resulting vector may serve as the normal in the point P.

In some models there are smooth parts and acute angles (point R on fig. 3 is located on such edge). In such a case it is not necessary to average the normals of neighboring polygons, but on the contrary – polygons on one side of the edge must be drawn with the use of one normal, and from the other – with the use of the other normal.

Analysis of methods for normalizing the vectors of normal

The scientific workers of the firm VIDIA received formula [4] for the approximate normalization of vectors of normals using one step of iteration of Newton - Rafson:

$$\vec{N}_n = \frac{\vec{N}}{3}(3 - \vec{N} \cdot \vec{N}) \cdot \tag{4}$$

The calculation in accordance with the given formula requires of operation of division, 6 operations of multiplication and three operations of addition.

Bigger absolute error of determining the orthogonal compounds of vector restrict the usage of formula for the tasks of covering.

According to the approach of P. lion the expression $1/\sqrt{\vec{N} \cdot \vec{N}}$, which is used for normalization of vector of normal $\vec{N}$, is distributed in Teilor series with the confine of the first three members. Then[5]:

$$\vec{N}_n = \vec{N}\left(1 - \frac{1}{2}\left(\left(\vec{N} \cdot \vec{N}\right) - 1\right) + \frac{3}{8}\left(\left(\vec{N} \cdot \vec{N}\right) - 1\right)^2\right). \tag{5}$$

Though the formula is suitable for realization in the hardware, it does not ensure the acceptable accuracy.

The interpolation of the single vectors of normals between the initial $\vec{N}_a$ and final $\vec{N}_b$ vectors, which have the single length, can be executed according to the formula[6]:

$$\vec{N}(w) = \vec{N}_a \frac{\sin((1-w)\psi)}{\sin\psi} + \vec{N}_b \frac{\sin(w\psi)}{\sin\psi}, \tag{6}$$

where $w \in [0,1]$, and ψ – angle between vectors of normals $\vec{N}_a$ и $\vec{N}_b$.

However, the significant drawback of the method is the necessity in calculation of trigonometric function sin, arccos for finding the specific vector of normal as well as for calculation of unknown parameters w and ψ. Apart from that, calculation $\vec{N}(w)$ stipulates for the execution of the division operation in the normalization cycle.

The work [7] suggests to use the principle of dichotomy for normalization of vectors in a way of consistent dividing in two the angle between the vectors of normals in the initial and final points of the line of triangle. The general formula looks like:

$$\vec{N}_{\left(1/2^{n+1}\right)} = \frac{\vec{N}_a + \vec{N}_{1/2^n}}{\sqrt{2 + z_{2^n}}}, \tag{7}$$

where $\vec{N}_a$ – initial vector; symbol $\left(1/2^n\right)$ means number of segments, received on n-iteration with the consistent dividing in two the segments of the line of rasterisation of triangle on condition that when $n = 0$ the segment equals the line of rasterisation ; $z_{2^n} = \sqrt{2\left(1 + \cos\frac{\psi}{2^{n-1}}\right)}$; ψ – angle between the vectors of normals $\vec{N}_a$ (initial) and $\vec{N}_b$ (final).

On each iteration for finding the denominator, onlz one operation of addition and one oprration on finding the square root is performed.

The advantage of the given method is $\frac{1}{\sqrt{2 + z_{2^n}}}$ possible to approximate with the series of Chebushev, which ensures insignificant errors in approximation. Using the multinomial of the first degree

$$\frac{1}{\sqrt{2+z_{2^n}}} \approx -0{,}07 \cdot z_{2^n} + 0{,}64 \tag{8}$$

the maximal absolute error of approximation does not exceed 0,0005, and the relative 0,12 %. The given formula is appropriate to use for the small graded screens, for which the triangles, which compose the surface of the three dimensional object, have the insignificant sizes.

Using the multinomial of the second degree

$$\frac{1}{\sqrt{2+z_{2^n}}} \approx 0{,}014 \cdot z^2{}_{2^n} - 0{,}119 \cdot z_{2^n} + 0{,}681 \tag{9}$$

the maximal absolute error of approximation does not exceed $2 \cdot 10^{-5}$, and the relative 0,004 %. Analysis shown that the use of this approximated formula the time of vector calculation $\vec{N}_{\left(\frac{1}{2^n}\right)}$ decreases 2,5 times in comparison with the classical realization.

The second method of normalization of vector of normal, suggested in [8], stipulates for using the quadratic interpolation on condition that the single vectors of normals are known in the initial and final points of the i-th line of triangle rasterisation. The intermediate values of vectors of normals in the line of triangle rasterisation are found according to the formula:

$$\vec{N}_{i,t} = \vec{G}_i \cdot t^2 + \vec{P}_i \cdot t + \vec{Q}_i . \tag{10}$$

Let $\vec{N}_{i,l}$, $\vec{N}_{i,p}$, $\vec{N}_{i,c}$ – correspondingly vectors of normals in the left, right and middle points of the line of triangle rasterisation. When $t=0$ $\vec{N}_{i,l} = \vec{Q}_i$. In the right point of the rasterisation line $t=1$, therefore $\vec{N}_{i,p} = \vec{G}_i + \vec{P}_i + \vec{Q}_i$. since in the middle point of the rasterisation line $t = 1/2$, then $N_{i,c} = \frac{\vec{G}_i}{4} + \frac{\vec{P}_i}{2} + \vec{Q}_i$. After the series of transformations it is possible to find the following expression for finding the vector of normal in the middle point:

$$\vec{N}_{i,c} = \frac{1}{\sqrt{2\left(1+\vec{N}_{i,l}\cdot\vec{N}_{i,p}\right)}}\left(\vec{N}_{i,l} + \vec{N}_{i,p}\right). \tag{11}$$

The advantage of the given method is the possibility to approximate the expression $\frac{1}{\sqrt{2\left(1+\vec{N}_{i,l}\cdot\vec{N}_{i,p}\right)}}$ by the polynomial of Chebushev of the second degree, which allows to decrease the time for calculation of the vector of normal in the middle point of the rasterisation line more than 2,5 times. Analysis showed that the software realization of the suggested method decreased the time for calculation of vector of normals for the middle triangle by 2,8 times in comparison with the classical realization.

For the formation of vectors of normals it is possibly to use the spherical angular interpolation [9] (fig. 4). The given method is appropriate for using during operation with angles on vectors of normals. The intermediate values of vectors of normals are to be found according to the formula:

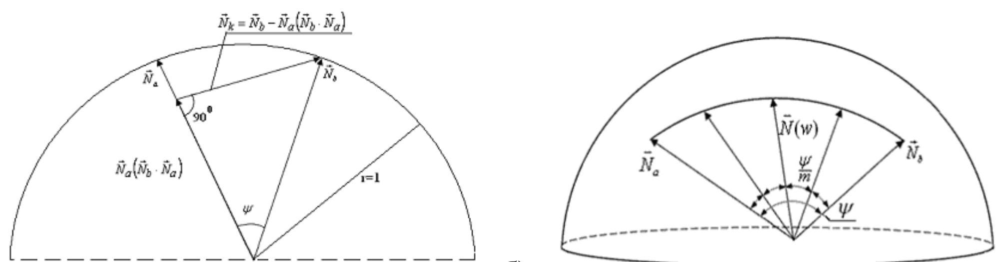
$$\vec{N}(t) = \vec{N}_a \cdot cos(\ t\cdot\varphi) + \vec{N}_k \cdot sin(t\cdot\varphi), \tag{12}$$

where t – number of pixel along the line of rasterisation, $t \in [0, l]$; $\vec{N}_a$ – initial vector; $\vec{N}_k = \frac{\vec{N}_b - \vec{N}_a(\vec{N}_b\cdot\vec{N}_a)}{\sqrt{1-\left(\vec{N}_b\cdot\vec{N}_a\right)^2}}$ – normalized vector of normal ; $\vec{N}_b$ – final vector; $\varphi = \psi / m$ – angle between the two connected vectors of normals, where $\psi = \arccos(\vec{N}_a \cdot \vec{N}_b)$ and m – length of line of the rasterisation.

The given formula may be written in the iteration form after some transformations:

$$\vec{N}(t+1)=2\vec{N}(t)\cdot\cos\varphi-\vec{N}(t-1). \tag{13}$$

The last formula allows to make a conclusion that the single vector of normal with spherical angular interpolation may be found through the previous two values.



а) б)

Fig. 4. Determination of vector $\vec{N}_k$ (a) and spherical angular interpolation of vectors of normals(б)

Analysis showed, that the software realization according to formula (13) allowed to decrease the calculation time of vectors of normals of the middle triangle by 1,7 times in comparison with the calculation of single vectors of normals by angular interpolation. Thus, it is possible to state that the improvement in operating of surface covering over is achieved.

Conclusion

The process of determination and normalization of vector of normal is a significant part of 3D images rendering process in the task of computer graphics. Many scientists dedicated their researches to the normalization of vectors of normal simplification. The analysis conducted in the given paper has revealed that the most efficient methods of normalization are those, which implement dichotomy principle and quadratic interpolation and spherical-angle interpolation, when working with vectors of normal angles. Described methods allow to improve calculation of vectors of normal time from 1,7 to 2,8 times comparing to classical normalization, which considerably increases 3D images rendering performance.

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