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SOLUTION OF INVERSE PROBLEM ON THE BASIS OF THE BASIS OF FUZZY LOGICAL EQUATIONS AND GENETIC ALGORITHM

This paper deals with restoration of inputs through observed outputs on the basis of fuzzy relational IF-THEN rules. The essence of the approach proposed consists in formulating and solving corresponding optimization problems which provide finding roots of fuzzy logical equations together with fuzzy model tuning on readily available experimental data.

***Key words**: inverse problem, fuzzy relational IF – THEN rules, solution of fuzzy logical equations, fuzzy model tuning, genetic algorithm.*

Introduction

Broad class of problems, emerging in engineering, medicine and other application domains relates to the class of inverse problems [1]. The essence of inverse problem is the following. The dependence Y=f(X), connecting vector X of non – observable parameters with vector Y of observable parameters is known. It is necessary to define by known values of vector Y the unknown values of vector X. Typical representative of inverse problem is the problem of medical and technical diagnostics, which is reduced to restoration of unknown reasons or diagnoses by observed consequences or symptoms. In cases, when the experience of experts is used for construction of reasons – consequence relations, it dependence between non – observable and observable parameters can be simulated by tools of fuzzy sets theory: fuzzy relations and fuzzy rules IF – THEN [2].

Analytical [3, 4] and numerical [5 – 7] techniques of inverse diagnostic problems solution based on fuzzy relations and Zade composition rule of conclusion are most elaborated ones. The given paper suggests the approach to the solution of inverse problem based on description of Y=f(X) dependence by means of fuzzy relational IF – THEN rules. Such rules allow to take into consideration simply and naturally complex combinations in cause – sequencial connection, which are simulated by fuzzy relations between separate terms [6, 7]. The problem is not only in solution of the system of fuzzy logic equations, which correspond to IF – THEN rules, but in the selection of such forms of membership functions of fuzzy terms and such weights of IF – THEN rules, which provide the maximum possible closeness between simulated and real outputs of the object. The essence of the approach proposed is in formulation and solution of corresponding optimization problems, which on one hand provide finding roots of fuzzy logical equations, and, on the other hand, - provide tuning of fuzzy model on available experimental data. For solution of the above – mentioned optimization problems we suggest to apply genetic algorithms.

1. Fuzzy model of object.

Connection «inputs $(x_1,...,x_n)$ – outputs $y_1,...,y_m)$» can be presented in the form of expert knowledge matrix (Table 1). Fuzzy database corresponds to this matrix:

IF $X = A_1$ with weight w_{11} OR … $X = A_K$ with weight w_{K1} THEN $Y = B_1$;

…

IF $X = A_1$ with weight w_{1Q} OR ... $X = A_K$ with weight w_{KQ} THEN $Y = B_Q$, (1)

where $A_l = \langle a_{1l},...,a_{nl}\rangle$ and $B_p = \langle b_{1p},...,b_{mp}\rangle$ – combinations of input and output terms, $l = \overline{1,K}$, $p = \overline{1,Q}$; a_{il} and b_{jp} – fuzzy terms, describing variables x_i and y_j in input and output combinations A_l and B_p; w_{lp} – weight of the rule, i.e. number in the interval [0, 1], reflecting the degree of A_l combination influence on occurring of B_p; K and Q – number of combinations of input and output terms.

The problem of inverse logical outputs is formed in the following way: by observed values of output variables $(y_1^*,...,y_m^*)$ the values of input values variables $(x_1^*,...,x_n^*)$ are to be restored. Restoration of inputs is reduced to solution of the system of fuzzy logic equations, which follows from (1):

$$\max_{l=\overline{1,K}}\left(\min\left[\mu^{A_l}(X), w_{11}\right]\right) = \min_{j=\overline{1,m}}\left[\mu^{b_{j1}}(y_j)\right]$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\max_{l=\overline{1,K}}\left(\min\left[\mu^{A_l}(X), w_{1Q}\right]\right) = \min_{j=\overline{1,m}}\left[\mu^{b_{jQ}}(y_j)\right], \quad (2)$$

where

$$\min_{i=\overline{1,n}}\left[\mu^{a_{i1}}(x_i)\right] = \mu^{A_1}$$

$$\ldots$$

$$\min_{i=\overline{1,n}}\left[\mu^{a_{iK}}(x_i)\right] = \mu^{A_K}. \quad (3)$$

Table 1

Fuzzy knowledge base

Then outputs					B_1	…	B_Q
				y_1	b_{11}	…	b_{1Q}
				…	…	…	…
IF inputs				y_m	b_{m1}	…	b_{mQ}
	x_1	…	x_n	Weight			
A_1	a_{11}	…	a_{n1}		w_{11}	…	w_{1Q}
	…	…	…		…	…	…
A_K	a_{1K}	…	a_{nK}		w_{K1}	…	w_{KQ}

Here $\mu^{a_{il}}(x_i)$ and $\mu^{b_{jp}}(y_j)$ – membership function of x_i and y_j variable to fuzzy terms a_{il} and b_{jp}; $\mu^{A_l}(X)$ – membership function of $\boldsymbol{X}$ vector to the combination of input terms A_l, $l=\overline{1,K}$.

Usage of fuzzy logic equations provides the presence of membership functions of fuzzy terms. We will use the following membership function of fuzzy term T:

$$\mu^T(u) = \frac{1}{1+\left(\frac{u-\beta}{\sigma}\right)^2}, \quad (4)$$

where β – coordinate of function maximum, $\mu^T(\beta)=1$; σ – parameter of concentration – tension.

Relations (2) – (4) define general form of fuzzy model of the object in the following way:

$$F_Y(\boldsymbol{X}, \boldsymbol{W}, \boldsymbol{B}_C, \boldsymbol{\Omega}_C) = \boldsymbol{\mu}^B(\boldsymbol{Y}, \boldsymbol{B}_E, \boldsymbol{\Omega}_E), \quad (5)$$

where $\boldsymbol{\mu}^B$ – vector of significance measures of output terms combination; $\boldsymbol{W}=(w_{11},...,w_{K1},...,w_{1Q},...,w_{KQ})$ – weight matrix; $\boldsymbol{B}_C=(\beta^{C_1},...,\beta^{C_N})$ and $\boldsymbol{\Omega}_C=(\sigma^{C_1},...,\sigma^{C_N})$ – vectors of β- and σ-parameters of membership functions of fuzzy terms of input variables C_1, …, C_N; $\boldsymbol{B}_E=(\beta^{E_1},...,\beta^{E_M})$ and $\boldsymbol{\Omega}_E=(\sigma^{E_1},...,\sigma^{E_M})$ – vectors of β- and σ-parameters of member ship functions of output variables E_1, … , E_M fuzzy terms; N and M – total number of fuzzy terms of input and output variables; F_Y – operator of "inputs - outputs" connection, corresponding to formulas (2) – (4).

2. Solution of the system of fuzzy logical equations

In accordance with the approach [5 – 7], the solution of the problem of the system of fuzzy logical equations (2) is formulated in the following manner. Find vector of significance measures of input terms $\boldsymbol{\mu}^C=(\mu^{C_1},...,\mu^{C_N})$, satisfying limitations $\mu^{C_I}\in[0,1]$, $\mathrm{I}=\overline{1,\mathrm{N}}$ and providing the least distance between modeled and observable significance measures of output terms combinations:

$$F=\sum_{p=1}^{Q}\left[\max_{l=\overline{1,K}}\left(\min\left(\mu^{A_l}(\boldsymbol{X}),w_{lp}\right)\right)-\mu^{B_p}(\boldsymbol{Y})\right]^2=\min_{\boldsymbol{\mu}^C}. \tag{6}$$

According to [3, 4] the set of solutions $S(\boldsymbol{W},\boldsymbol{\mu}^B)$ of the system (2) is defined by single maximum solution $\overline{\boldsymbol{\mu}}^A$ and by the set of minimum solutions $S^*(\boldsymbol{W},\boldsymbol{\mu}^B)=\left\{\underline{\boldsymbol{\mu}}_t^A,\ t=\overline{1,T}\right\}$:

$$S(\boldsymbol{W},\boldsymbol{\mu}^B)=\bigcup_{\underline{\boldsymbol{\mu}}_t^A\in S^*}\left[\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right]. \tag{7}$$

Here $\overline{\boldsymbol{\mu}}^A=(\overline{\mu}^{A_1},...,\overline{\mu}^{A_K})$ and $\underline{\boldsymbol{\mu}}_t^A=(\underline{\mu}_t^{A_1},...,\underline{\mu}_t^{A_K})$ – are vectors of upper and lower boundaries of significance measures of input combinations A_l, where union operation is performed over $\underline{\boldsymbol{\mu}}_t^A\in S^*(\boldsymbol{W},\boldsymbol{\mu}^B)$.

Each interval solution $\left[\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right]$, $t=\overline{1,T}$ of the system (2) corresponds to the set of solutions $D_t\left(\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right)$ of the system (3), which is defined by single minimum solution $\underline{\boldsymbol{\mu}}_t^C$ and by the set of maximum solutions $D_t^*\left(\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right)=\left\{\overline{\boldsymbol{\mu}}_{th}^C,h=\overline{1,H_t}\right\}$:

$$D_t\left(\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right)=\bigcup_{\overline{\boldsymbol{\mu}}_{th}^C\in D_t^*}\left[\underline{\boldsymbol{\mu}}_t^C,\overline{\boldsymbol{\mu}}_{th}^C\right]. \tag{8}$$

Here $\underline{\boldsymbol{\mu}}_t^C=(\underline{\mu}_t^{C_1},...,\underline{\mu}_t^{C_N})$ and $\overline{\boldsymbol{\mu}}_{th}^C=(\overline{\mu}_{th}^{C_1},...,\overline{\mu}_{th}^{C_N})$ – are vectors of upper and lower boundaries of membership degrees of inputs to terms C_I, where union operation is performed over all $\overline{\boldsymbol{\mu}}_{th}^C\in D_t^*\left(\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right)$.

Based on relations (7) and (8) the set of the system (2) solutions is defined as:

$$\widetilde{D}(\boldsymbol{W},\boldsymbol{\mu}^A,\boldsymbol{\mu}^B)=\bigcup_{\underline{\boldsymbol{\mu}}_t^A\in S^*(\boldsymbol{W},\boldsymbol{\mu}^B)}D_t\left(\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A\right). \tag{9}$$

Formation of solutions (9) set starts with the search of zero solution $\boldsymbol{\mu}_0^C=(\mu_0^{C_1},...,\mu_0^{C_N})$ of optimization problem (6) applying [5 – 7]. Modified fuzzy vector $\boldsymbol{\mu}_0^B=(\mu_0^{B_1},...,\mu_0^{B_Q})$ which provides analytical solution of the system of fuzzy logic equations (2) and (3), corresponds to zero solution of $\boldsymbol{\mu}_0^C$. Formation of the set of solution $S(\boldsymbol{W},\boldsymbol{\mu}_0^B)$ for modified vector $\boldsymbol{\mu}_0^B$ is performed with the help of accurate analytical methods, realized in software applications MATLAB [4].

3. Tuning of fuzzy model

Let teaching sample be set in the form of L pairs of experimental data: $\left\langle\hat{\boldsymbol{X}}_k,\hat{\boldsymbol{Y}}_k\right\rangle$ $k=\overline{1,L}$, where $\hat{\boldsymbol{X}}_k=\left(\hat{x}_1^k,...,\hat{x}_n^k\right)$ and $\hat{\boldsymbol{Y}}_k=\left(\hat{y}_1^k,...,\hat{y}_m^k\right)$ - are vectors of input and output variables values in the experiment with k number. The essence of tuning is the selection of such zero solution of

$\boldsymbol{\mu}_0^C(\hat{x}_1^k,...,\hat{x}_n^k)$ inverse problem which minimize the criterion (6) for all the points of teaching sample.

$$\sum_{\mathrm{k}=1}^{\mathrm{L}}\left[F_Y(\boldsymbol{\mu}_0^C(\hat{x}_1^k,...,\hat{x}_n^k))-\hat{\boldsymbol{\mu}}^B(\hat{y}_1^k,...,\hat{y}_m^k)\right]^2=\min.$$

In other words, it is necessary to find out such vector of $\boldsymbol{W}$ weights and such vectors of parameters of membership functions $\boldsymbol{B}_C$ $\boldsymbol{\Omega}_C$ $\boldsymbol{B}_E$ $\boldsymbol{\Omega}_E$, which provide minimum distance between modeled and experimental vectors of significance measures of output terms combinations :

$$\sum_{\mathrm{k}=1}^{\mathrm{L}}\left[F_Y(\hat{\boldsymbol{X}}_{\mathrm{k}},\boldsymbol{W},\boldsymbol{B}_{\mathrm{C}},\boldsymbol{\Omega}_{\mathrm{C}})-\hat{\boldsymbol{\mu}}^B(\hat{\boldsymbol{Y}}_k,\boldsymbol{B}_{\mathrm{E}},\boldsymbol{\Omega}_{\mathrm{E}})\right]^2=\min_{\boldsymbol{W},\boldsymbol{B}_C,\boldsymbol{\Omega}_C,\boldsymbol{B}_E,\boldsymbol{\Omega}_E} \tag{10}$$

In genetic algorithm the solution of chromosom optimization problem (10) is defined as the vector of $\boldsymbol{W}$, $\boldsymbol{B}_C$, $\boldsymbol{\Omega}_C$, $\boldsymbol{B}_E$, $\boldsymbol{\Omega}_E$ parameters codes, and correspondence function is built on criterion base(10).

4. Computer-based experiment

The goal of experiment was to restore the reference model “two inputs (x_1,x_2) – two outputs (y_1,y_2)” shown in Fig 1:

$$y_1=f_1(x_1,x_2)=((2z_1-0{,}9)\ (7z_1-1)\ (17z_2-19)\ (15z_2-2))/10\ ,\ y_2=f_2(x_1,x_2)=-y_1+3.4\ ,$$

where $z_1=\left((x_1-2{,}9)^2+(x_2-2{,}9)^2\right)/39$, $z_2=(x_1-3{,}1)^2+(x_2-3{,}1)^2/41$.

Fuzzy rules IF – THEN from Table 2 correspond to this model, where inputs and outputs were described by fuzzy terms.

Low C_1 *(L)*, *average* C_2 *(A)*, *High* C_3 *(H)* for x_1; C_4 *(L)*, C_5 *(A)*, C_6 *(H)* for x_2; E_1*=higher than the low*(*hL*), E_2*= lower than average (lA),* E_3*=High (H)* for y_1; E_4*=Low (L)*, E_5*=higher than average (hA)*, E_6*=lower than the high (lH)* for y_2.

Weight matrix was formed on the basis of pair comparisons [6, 7]. Results of fuzzy model tuning are given in Table 3. Results of the solution of inverse output problem after tuning are shown in Fig 2. The same figure shows membership functions of fuzzy terms of input and output variables after tuning.

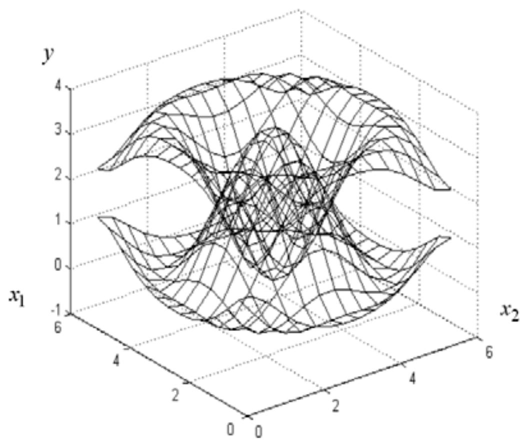


Fig. 1. Model - generator «Inputs– outputs»

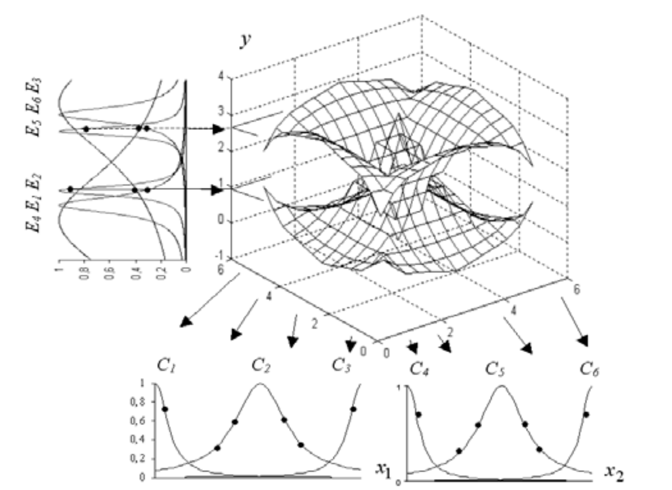


Fig. 2. Solution inverse output problem after tuning

Table 3

Parameters of membership functions of fuzzy terms after tuning

Parameter	Fuzzy terms of input variables						Parameter	Fuzzy terms of output variables					
	C_1	C_2	C_3	C_4	C_5	C_6		E_1	E_2	E_3	E_4	E_5	E_6
β -	0.03	3.04	5.97	0.02	3.05	5.96	β -	0.52	0.91	3.35	0.10	2.57	3.03
σ -	0.41	0.82	0.39	0.43	0.9	0.4	σ -	0.28	0.16	1.95	1.93	0.14	0.26

Fuzzy logic equations after tuning have form:

$$(\mu^{A_1} \wedge 0.60) \vee (\mu^{A_2} \wedge 1.00) \vee (\mu^{A_3} \wedge 0.60) \vee (\mu^{A_4} \wedge 1.00) \vee (\mu^{A_5} \wedge 0.06) \vee$$
$$\vee (\mu^{A_6} \wedge 1.00) \vee (\mu^{A_7} \wedge 0.60) \vee (\mu^{A_8} \wedge 1.00) \vee (\mu^{A_9} \wedge 0.60) = \mu^{E_1} \wedge \mu^{E_6}$$
$$(\mu^{A_1} \wedge 1.00) \vee (\mu^{A_2} \wedge 0.52) \vee (\mu^{A_3} \wedge 1.00) \vee (\mu^{A_4} \wedge 0.52) \vee (\mu^{A_5} \wedge 0.19) \vee$$
$$\vee (\mu^{A_6} \wedge 0.52) \vee (\mu^{A_7} \wedge 1.00) \vee (\mu^{A_8} \wedge 0.52) \vee (\mu^{A_9} \wedge 1.00) = \mu^{E_2} \wedge \mu^{E_5}$$
$$(\mu^{A_1} \wedge 0.34) \vee (\mu^{A_2} \wedge 0.06) \vee (\mu^{A_3} \wedge 0.34) \vee (\mu^{A_4} \wedge 0.06) \vee (\mu^{A_5} \wedge 1.00) \vee$$
$$\vee (\mu^{A_6} \wedge 0.06) \vee (\mu^{A_7} \wedge 0.34) \vee (\mu^{A_8} \wedge 0.06) \vee (\mu^{A_9} \wedge 0.34) = \mu^{E_3} \wedge \mu^{E_4}, \quad (11)$$

where

$$\mu^{C_1} \wedge \mu^{C_4} = \mu^{A_1}$$
$$\mu^{C_2} \wedge \mu^{C_4} = \mu^{A_2}$$
$$\mu^{C_3} \wedge \mu^{C_4} = \mu^{A_3}$$
$$\mu^{C_1} \wedge \mu^{C_5} = \mu^{A_4}$$
$$\mu^{C_2} \wedge \mu^{C_5} = \mu^{A_5}$$
$$\mu^{C_3} \wedge \mu^{C_5} = \mu^{A_6}$$
$$\mu^{C_1} \wedge \mu^{C_6} = \mu^{A_7}$$
$$\mu^{C_2} \wedge \mu^{C_6} = \mu^{A_8}$$
$$\mu^{C_3} \wedge \mu^{C_6} = \mu^{A_9}. \quad (12)$$

Table 2.

Fuzzy matrix of knowledge for model - reference

IF inputs			Then outputs	B_1	B_2	B_3
			y_1	lA	hL	H
			y_2	hA	lH	L
	x_1	x_2	**weight**			
A_1, A_3, A_7, A_9	***L (H)***	***L(H)***		0.67	1.00	0.44
A_2, A_4	*A (L)*	*L(A)*		1.00	0.50	0.11
A_6, A_8	*H (A)*	*A(H)*				
A_5	*A*	*A*		0.11	0.17	1.00

Let the given values of outputs be y_1^*=0.95 and y_2^*=2.65. For these values with the help of membership functions in Fig 2 measures of significance of output terms were defined.: $\mu^{E_1}(y_1^*)$=0.30; $\mu^{E_2}(y_1^*)$=0.94; $\mu^{E_3}(y_1^*)$=0.40; $\mu^{E_4}(y_2^*)$=0.36; $\mu^{E_5}(y_2^*)$=0.75; $\mu^{E_6}(y_2^*)$=0.32.

Vector of significance measures of output terms was:

$$\boldsymbol{\mu}^B(Y^*) = (\mu^{B_1} = 0.30;\ \mu^{B_2} = 0.75;\ \mu^{B_3} = 0.36).$$

By means genetic algorithm zero solution was obtained:

$$\boldsymbol{\mu}_0^C = (\mu_0^{C_1} = 0.91, \mu_0^{C_2} = 0.43, \mu_0^{C_3} = 0.80, \mu_0^{C_4} = 0.75, \mu_0^{C_5} = 0.36, \mu_0^{C_6} = 0.75),$$

corresponded by modified fuzzy vector

$$\boldsymbol{\mu}_0^B = (\mu_0^{B_1} = 0.60, \mu_0^{B_2} = 0.75, \mu_0^{B_3} = 0.36),$$

i.e. value of optimization criterion (9) was F=0.0900.

Applying MATLAB – application ***solve_flse.m*** [4] for modified vector $\boldsymbol{\mu}_0^B$ the set of solutions $S(\boldsymbol{W}, \boldsymbol{\mu}_0^B)$ system (11) was formed, which is defined by single maximum solution $\overline{\boldsymbol{\mu}}^A$ and four minimum solutions $S^* = \{\underline{\boldsymbol{\mu}}_t^A, t = \overline{1, 4}\}$:

$$S(\boldsymbol{W}, \boldsymbol{\mu}_0^B) = \bigcup_{t=1,\dots,4} [\underline{\boldsymbol{\mu}}_t^A, \overline{\boldsymbol{\mu}}^A] = \{\mu^{A_2} = \mu^{A_4} = \mu^{A_6} = \mu^{A_8} \in [0, 0.6],\ \mu^{A_5} = 0.36\} \cap$$
$$\{\{\mu^{A_1} = 0.75,\ \mu^{A_3} = \mu^{A_7} = \mu^{A_9} \in [0, 0.75]\} \cup \{\mu^{A_3} = 0.75,\ \mu^{A_1} = \mu^{A_7} = \mu^{A_9} \in [0, 0.75]\} \cup$$
$$\{\mu^{A_7} = 0.75,\ \mu^{A_1} = \mu^{A_3} = \mu^{A_9} \in [0, 0.75]\} \cup \{\mu^{A_9} = 0.75,\ \mu^{A_1} = \mu^{A_3} = \mu^{A_7} \in [0, 0.75]\}\}. \quad (13)$$

For intervals (13) with the help of MATLAB-application ***solve_flse.m*** [4] sets of solutions $D_t(\underline{\boldsymbol{\mu}}_t^A, \overline{\boldsymbol{\mu}}^A)$, $t = \overline{1, 4}$ of the system were formed (12).

The set of solution $D_1(\underline{\boldsymbol{\mu}}_1^A, \overline{\boldsymbol{\mu}}^A)$ is defined by single minimum solution $\underline{\boldsymbol{\mu}}_1^C$ and four maximum solutions $D_1^* = \{\overline{\boldsymbol{\mu}}_{1h}^C, h = \overline{1, 4}\}$

$$D_1(\underline{\boldsymbol{\mu}}_1^A, \overline{\boldsymbol{\mu}}^A) = \bigcup_{h=1,\dots,4} [\underline{\boldsymbol{\mu}}_1^C, \overline{\boldsymbol{\mu}}_{1h}^C] = \{\mu^{C_3} \in [0, 0.75],\ \mu^{C_6} \in [0, 0.75]\} \cap$$
$$\{\{\{\mu^{C_1} = 0.75,\ \mu^{C_4} \in [0.75, 1.0]\} \cup \{\mu^{C_1} \in [0.75, 1.0],\ \mu^{C_4} = 0.75\}\} \cap$$
$$\{\{\mu^{C_2} = 0.36,\ \mu^{C_5} \in [0.36, 0.6]\} \cup \{\mu^{C_2} \in [0.36, 0.6],\ \mu^{C_5} = 0.36\}\}\}.$$

The set of solutions $D_2(\underline{\boldsymbol{\mu}}_2^A, \overline{\boldsymbol{\mu}}^A)$ is defined by single minimum solution $\underline{\boldsymbol{\mu}}_2^C$ and four maximum solutions $D_2^* = \{\overline{\boldsymbol{\mu}}_{2h}^C, h = \overline{1, 4}\}$

$$D_2(\underline{\boldsymbol{\mu}}_2^A, \overline{\boldsymbol{\mu}}^A) = \bigcup_{h=1,\dots,4} [\underline{\boldsymbol{\mu}}_2^C, \overline{\boldsymbol{\mu}}_{2h}^C] = \{\mu^{C_1} \in [0, 0.75],\ \mu^{C_6} \in [0, 0.75]\} \cap$$
$$\{\{\{\mu^{C_3} = 0.75,\ \mu^{C_4} \in [0.75, 1.0]\} \cup \{\mu^{C_3} \in [0.75, 1.0],\ \mu^{C_4} = 0.75\}\} \cap$$
$$\{\{\mu^{C_2} = 0.36,\ \mu^{C_5} \in [0.36, 0.6]\} \cup \{\mu^{C_2} \in [0.36, 0.6],\ \mu^{C_5} = 0.36\}\}\}.$$

The set of solutions $D_3(\underline{\boldsymbol{\mu}}_3^A, \overline{\boldsymbol{\mu}}^A)$ is defined by single minimum solution $\underline{\boldsymbol{\mu}}_3^C$ and four maximum solutions $D_3^* = \{\overline{\boldsymbol{\mu}}_{3h}^C, h = \overline{1, 4}\}$

$$D_3(\underline{\boldsymbol{\mu}}_3^A, \overline{\boldsymbol{\mu}}^A) = \bigcup_{h=1,\dots,4} [\underline{\boldsymbol{\mu}}_3^C, \overline{\boldsymbol{\mu}}_{3h}^C] = \{\mu^{C_3} \in [0, 0.75],\ \mu^{C_4} \in [0, 0.75]\} \cap$$
$$\{\{\{\mu^{C_1} = 0.75,\ \mu^{C_6} \in [0.75, 1.0]\} \cup \{\mu^{C_1} \in [0.75, 1.0],\ \mu^{C_6} = 0.75\}\} \cap$$
$$\{\{\mu^{C_2} = 0.36,\ \mu^{C_5} \in [0.36, 0.6]\} \cup \{\mu^{C_2} \in [0.36, 0.6],\ \mu^{C_5} = 0.36\}\}\}.$$

The set of solutions $D_4(\underline{\boldsymbol{\mu}}_4^A, \overline{\boldsymbol{\mu}}^A)$ is defined by single minimum solution $\underline{\boldsymbol{\mu}}_4^C$ and four maximum solutions $D_4^* = \{\overline{\boldsymbol{\mu}}_{4h}^C, h = \overline{1, 4}\}$

$$D_4(\underline{\boldsymbol{\mu}}_4^A, \overline{\boldsymbol{\mu}}^A) = \bigcup_{h=1,\dots,4} [\underline{\boldsymbol{\mu}}_4^C, \overline{\boldsymbol{\mu}}_{4h}^C] = \{\mu^{C_1} \in [0, 0.75],\ \mu^{C_4} \in [0, 0.75]\} \cap$$

$$\{\{\{\mu^{C_3}=0.75,\ \mu^{C_6}\in[0.75,\ 1.0]\}\cup\{\mu^{C_3}\in[0.75,\ 1.0],\ \mu^{C_6}=0.75\}\}\cap$$
$$\{\{\mu^{C_2}=0.36,\ \mu^{C_5}\in[0.36,\ 0.6]\}\cup\{\mu^{C_2}\in[0.36,\ 0.6],\ \mu^{C_5}=0.36\}\}\}.$$

Thus, the solution of the system of fuzzy logic equations (11) has been form:

$$\widetilde{D}(\boldsymbol{W},\boldsymbol{\mu}^A,\boldsymbol{\mu}^B)=\bigcup_{t=1,\dots,4} D_t(\underline{\boldsymbol{\mu}}_t^A,\overline{\boldsymbol{\mu}}^A) \qquad (14)$$

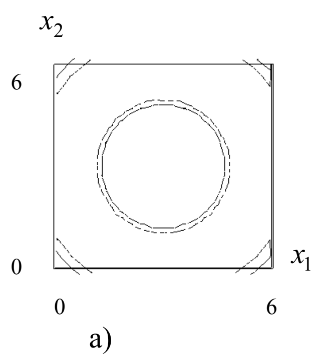
For each interval in the solution (14), applying membership functions in Fig 2, the intervals of input variables values can be defined:

$x_1^*\in[0,\ 0.27]$ or $x_1^*\in[0.27,\ 6.0]$ for C_1; $x_1^*\in[1.95,\ 2.37]$ or $x_1^*\in[3.71,\ 4.13]$ for C_2;

$x_1^*\in[0,\ 5.74]$ or $x_1^*\in[5.74,\ 6.0]$ for C_3; $x_2^*\in[0,\ 0.27]$ or $x_2^*\in[0.27,\ 6.0]$ for C_4;

$x_2^*\in[1.85,\ 2.32]$ or $x_2^*\in[3.78,\ 4.25]$ for C_5; $x_2^*\in[0,\ 5.73]$ or $x_2^*\in[5.73,\ 6.0]$ for C_6.

Restoration of inputs set for y_1^*=0.95 and y_2^*=2.65 is shown in Fig 2. Markers label value of membership degrees of input and outputs to fuzzy terms $C_1\div C_6$ and $E_1\div E_6$. Comparison of reference and restored level lines for y_1^*=0.95 and y_2^*=2.65 is shown in Fig 3. Increase of approximation accuracy is possible due to increase of the number of fuzzy terms, that, in its turn, allows to increase the number of polygon sides, approximating the circle.



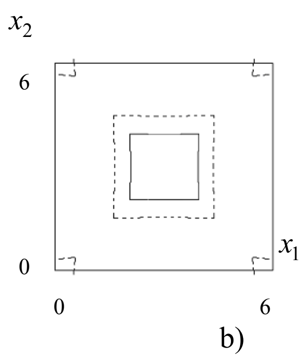


Fig. 3. Comparison of reference (a) and restored (б) level lines for y_1^*=0.95 (____) and y_2^*=2.65 (_ _ _)

Conclusions

The given paper suggests the approach to the solution of inverse problem based on description of relation between non – observable and observable parameters of the object on the basis of fuzzy relational IF- THEN rules. Restoration of inputs by observable outputs is carried out by means of solution of the system of fuzzy logic equations, corresponding to IF – THEN rules and tuning fuzzy model tuning on available experimental data. For the solution of optimization problems genetic algorithms are proposed. The efficiency of suggested models and algorithm is proved by computer experiment. The considered approach can find application in engineering, medicine, economy and other fields where there is the necessity of experimental data interpretation.

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