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MATHEMATICAL MODEL OF REGULATING PROCESS BY BALANCING TRANSFORMER

Mathematical control model of balancing transformer with winding connection circuit «triangle – zigzag» at longitudinal asymmetry, allowing to attain pinpoint accuracy of voltage balancing and considering influence of common industrial load is synthesized.

Keywords: *balancing transformer, "triangle – zigzag", accuracy of balancing, load current, transfer ratio, three-phase regulating, factor of voltage asymmetry of reverse sequence.*

State of art and problem set-up

One of the important characteristics of power quality is longitudinal asymmetry of voltages. In [1 - 3] balancing transformers with winding connection circuit "triangle – star" as tools of voltage balancing were investigated. Mathematical models offered in these research, allowed to obtain a number of advantages due to the usage of balancing transformer for voltage quality improvement in electric-power supply systems in comparison with other types of balancing installations. However drawback of balancing transformer with winding connection circuit "triangle - star" usage is emerging of zero-sequence voltage as a result of realization of phase-by-phase regulating and, accordingly, deviations of voltages that reduces efficiency of balancing. More effective, in our opinion, is use of balancing transformers with winding connection circuit «triangle – zigzag» (Fig.1) which do not possess specified drawbacks.

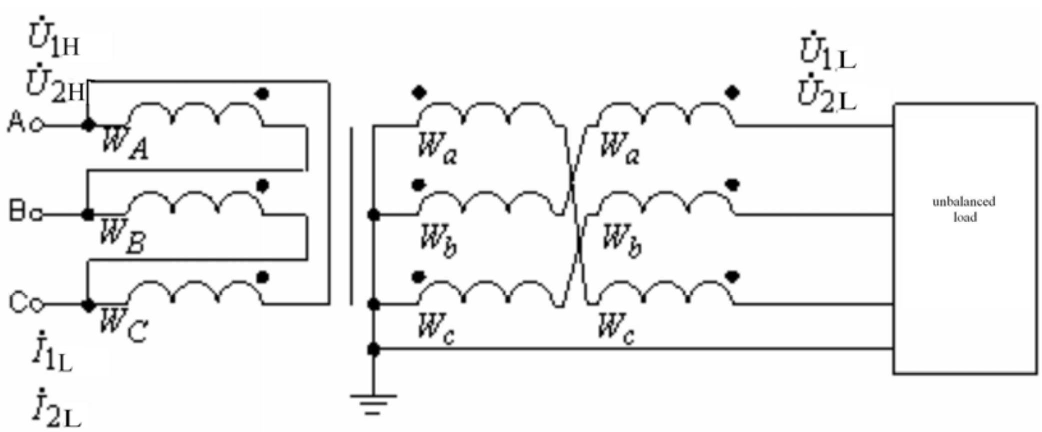

Fig. 1. Circuit of winding connection of balancing transformer

Simplified condition of voltage balancing in case of usage of balancing transformer with winding connection circuit «triangle -zigzag» is substantiated in [4]:

$$\left(\dot{U}_{1B} - \dot{I}_{1Л}\frac{\underline{Z}_K}{3}\right)\cdot\underline{K}_2 + \left(\dot{U}_{2B} - \dot{I}_{2Л}\frac{\underline{Z}_K}{3}\right)\cdot\underline{K}_1 e^{-j60°} = 0, \qquad (1)$$

where $\underline{K}_1 = -\frac{1}{3}(k_A + k_B + k_C)$; $\underline{K}_2 = -\frac{1}{3}\left(k_A + ak_B + a^2k_C\right)$ – complex transfer ratios of balancing transformer, k_A, k_B, k_C - phase transfer ratios of transformer which consider half of loops of secondary winding connected in "zigzag"; $\dot{U}_{1B}$, $\dot{U}_{2B}$ – complex linear voltages of direct and reverse sequences of primary winding of transformer; $\dot{I}_{1L}$, $\dot{I}_{2L}$ – complex linear currents of direct and reverse sequences at balancing transformer high-voltage side; $\underline{Z}_{SC}$ – transformer short circuit impedance.

In [4] the conditions of three-phase regulation by balancing transformer also have been obtained:

$$\begin{cases}(1+\Delta k_A)a+(1+\Delta k_B)b+(1+\Delta k_C)c=0;\\(1+\Delta k_A)d+(1+\Delta k_B)f+(1+\Delta k_C)g=0;\\\Delta k_A+\Delta k_B+\Delta k_C=\Delta k/3,\end{cases} \tag{2}$$

where Δk_A, Δk_B, Δk_C – increments of transfer ratios to provide three-phase regulation; Δk – increment of mean value of transfer ratio, ensuring decrease of voltage deviations;

$$a=u_1'+\frac{1}{2}u_2'+\frac{\sqrt{3}}{2}u_2'';\ b=-\frac{1}{2}u_1'-\frac{\sqrt{3}}{2}u_1''+\frac{1}{2}u_2'+\frac{\sqrt{3}}{2}u_2'';\ c=-\frac{1}{2}u_1'+\frac{\sqrt{3}}{2}u_1''+\frac{1}{2}u_2'+\frac{\sqrt{3}}{2}u_2'';$$

$$d=u_1''+\frac{1}{2}u_2''-\frac{\sqrt{3}}{2}u_2';\ f=-\frac{1}{2}u_1''+\frac{\sqrt{3}}{2}u_1'+\frac{1}{2}u_2''-\frac{\sqrt{3}}{2}u_2';\ g=-\frac{1}{2}u_1''-\frac{\sqrt{3}}{2}u_1'+\frac{1}{2}u_2''-\frac{\sqrt{3}}{2}u_2';$$

$u_1'+ju_1''=\dot{U}_{1B}-\dot{I}_{1Л}\frac{\underline{Z}_K}{3}$, $u_2'+ju_2''=\dot{U}_{2B}-\dot{I}_{2Л}\frac{\underline{Z}_K}{3}$ - value of measured magnitudes.

Three-phase regulating which is carried out accordingly (2), meets such criteria:

$$\dot{U}_{2H}\rightarrow\min;\ \Delta U_{\max(H)}\rightarrow\min,$$

where $\dot{U}_{2L}$ – voltage of reverse sequence on low side of transformer; $\Delta U_{\max(H)}$ – maximum deviation of phase voltage on secondary winding.

The condition (1) is obtained for simplified equivalent circuit of transformer in which the effect of transformation ratios of phases on values of primary winding currents is neglected.

In the article the problem of synthesis of specified mathematical models of balancing transformer control with winding connection circuit «triangle – zigzag» is solved, that is ensured with the account of asymmetrical loading currents influence. Of special interest is also evaluation of balancing mistakes in case of use of conditions (1) and (2).

Substantiation of the results

From the analysis of expression for voltage of secondary winding of transformer with connection circuit "triangle - star"(at the example of phase A voltage) taking into account magnetization circuit [5] we have:

$$\frac{\dot{U}_a}{k}=\dot{E}_{AB}-k\dot{I}_a(r_2+jx_2), \tag{3}$$

where $\dot{U}_a$ – complex phase voltage of secondary winding of transformer; $k=W_a/W_A$ – transformation ratio; $\dot{E}_{AB}=m\cdot\dot{U}_{AB}$ – complex generated voltage of primary winding (here $m\approx1$); $\dot{I}_a$ – complex phase current of secondary winding; r_2, x_2 – resulted active and reactive resistances of secondary winding.

On the basis of expressions for transformer with connection circuit "triangle - star", having used superposition principle, we can obtain expressions for phase voltages of transformer with winding connection circuit "triangle - zigzag"

$$\dot{U}_A=\left(mk_A\dot{U}_{AB}-\frac{1}{2}k_A^2\underline{Z}_K\dot{I}_a\right)-\left(mk_B\dot{U}_{BC}-\frac{1}{2}k_B^2\underline{Z}_K\dot{I}_b\right);$$

$$\dot{U}_B=\left(mk_B\dot{U}_{BC}-\frac{1}{2}k_B^2\underline{Z}_K\dot{I}_b\right)-\left(mk_C\dot{U}_{CA}-\frac{1}{2}k_C^2\underline{Z}_K\dot{I}_c\right);$$

$$\dot{U}_C = \left(mk_C \dot{U}_{CA} - \frac{1}{2} k_C^2 \underline{Z}_K \dot{I}_c \right) - \left(mk_A \dot{U}_{AB} - \frac{1}{2} k_A^2 \underline{Z}_K \dot{I}_a \right).$$

After carrying out mathematical conversions condition of voltage balancing by means of balancing transformer with connection circuit "triangle - zigzag" taking into account load current impact (influence of phases transformation ratios on values of primary winding currents) will be:

$$m\dot{U}_{1B}\underline{K}_2 + m\dot{U}_{2B}\underline{K}_1 e^{-j60°} + \frac{1}{6\cdot\sqrt{3}}\underline{Z}_K \left[k_A I_{AB} e^{-j30°} + k_B I_{BC} e^{-j150°} + k_C I_{CA} e^{j90°} \right] = 0. \quad (4)$$

The analysis (4) has allowed to obtain conditions of three-phase regulation of balancing transformer:

$$\begin{cases} (1+\Delta k_A)(-h' + k' + l') + (1+\Delta k_B)\left(\frac{1}{2}h' + \frac{\sqrt{3}}{2}h'' + k' + n'\right) + (1+\Delta k_C)\left(\frac{1}{2}h' + \frac{\sqrt{3}}{2}h'' + k' + p'\right) = 0; \\ (1+\Delta k_A)(-h'' + k'' + l'') + (1+\Delta k_B)\left(-\frac{\sqrt{3}}{2}h' + \frac{1}{2}h'' + k'' + n''\right) + (1+\Delta k_C)\left(\frac{\sqrt{3}}{2}h' + \frac{1}{2}h'' + k'' + p''\right) = 0; \\ \Delta k_A + \Delta k_B + \Delta k_C = \Delta k, \end{cases} \quad (5)$$

where $h' = \mathrm{Re}(m\dot{U}_{1B})$; $h'' = \mathrm{Im}(m\dot{U}_{1B})$; $k' = \mathrm{Re}(m\dot{U}_{2B}e^{j120°})$; $k'' = \mathrm{Im}(m\dot{U}_{2B}e^{j120°})$;

$$l' = \mathrm{Re}\left(\frac{\sqrt{3}}{6}\underline{Z}_K I_{AB} e^{j30°}\right); \; l'' = \mathrm{Im}\left(\frac{\sqrt{3}}{6}\underline{Z}_K I_{AB} e^{j30°}\right);$$

$$n' = \mathrm{Re}\left(\frac{\sqrt{3}}{6}\underline{Z}_K I_{BC} e^{-j150°}\right); \; n'' = \mathrm{Im}\left(\frac{\sqrt{3}}{6}\underline{Z}_K I_{BC} e^{-j150°}\right);$$

$$p' = \mathrm{Re}\left(\frac{\sqrt{3}}{6}\underline{Z}_K I_{CA} e^{j90°}\right); \; p'' = \mathrm{Im}\left(\frac{\sqrt{3}}{6}\underline{Z}_K I_{CA} e^{j90°}\right).$$

In Fig. 2 graphs of dependences of voltages asymmetry factor of reverse sequence k_{2U} while using three-phase regulation of balancing transformer with winding connection circuit "triangle - zigzag" are shown. Curve 1 characterizes dependence of factor k_{2U} which will be set after balancing according to condition (5) at load current which varies from null to nominal. Curves 2 and 3 show dependences k_{2U} which will be set after balancing according to conditions (2) at load current which is equal accordingly 50 % and 100 % of rated value.

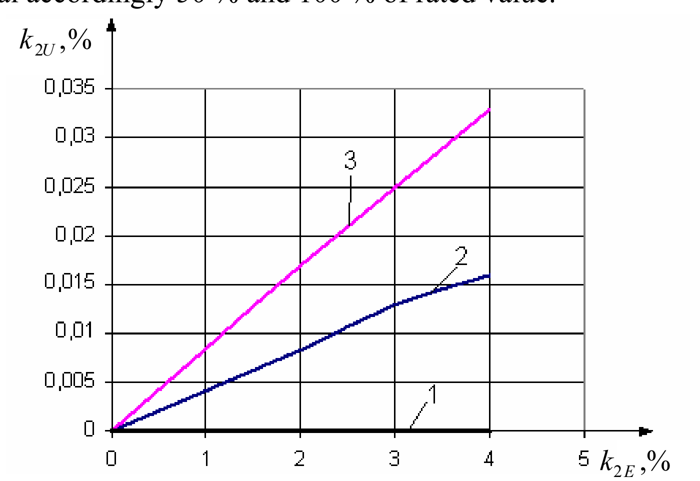

Fig. 2. Dependences of voltages asymmetry factor of reverse sequence on different conditions of balancing

As it follows from the graphs, use of conditions which take into account influence of load current, ensures more exact symmetrizing of voltages. At usage of balancing conditions (2), inaccuracy of balancing increases with load current increase. At the same time use of balancing conditions (2) leads to insignificant increase of balancing errors, and therefore the condition (2) (as more simple is recommended for practical implementation).

Conclusions

The mathematical model of control by balancing transformer with winding connection circuit «triangle - zigzag» is synthesized at longitudinal asymmetry of voltage, allowing to attain pinpoint accuracy of voltage balancing and taking into account the influence of common industrial loading is synthesized. Mathematical modelling shows, that account of asymmetrical loading slightly influences the accuracy of balancing.

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