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# COMPLEX OF OPERATION MODELS FOR MODELING AND OPTIMIZATION OF CREDIT STRATEGIES

*The problem of optimum processes modeling of production systems development taking into account the use of credits is considered and solved. Optimum strategies of crediting and credits reimbursement are obtained. Demand function for the case of optimal control of development are obtained.*

***Keywords:*** *distributed system, optimum development, producer, demand, credits, optimum credit strategy of development, operation model.*

**Problem set-up.** Efficiency of crediting processes is a condition of survival and successful development, both for borrowers and creditors – banks. We will consider only credits to develop production from the position of the borrower- manufacturer. In this case task of determination of crediting rate and credits distribution between directions of enterprise development or between products which enterprise manufactures and plans to manufacture , is the task of optimum management of nonlinear dynamic system due to own and external resources. Financial and legal aspects are only transaction charges on realization of crediting process. Optimization of crediting process today is a condition of survival not only of individual producer or a bank, but also industry and national economy as a whole.

**Unsolved parts of the problem.** In the available literature there are no mathematical models to identify the best strategies for credit, reimbursement of credits and calculation of risks. The first reason of the "vacuum" of publications is confidentiality of corporate science, second – "nobody territory" and absence of specialists on the system analysis, management and programming. In the available literature on management projects there are only verbal receipts and inadequate models. The reason is that we are dealing with complex variational task which requires an integrated approach: "programming + theory of optimum management + economy". A collateral problem is non -compatibility of standards of publications with the standards of mathematical packages and packages for modeling. Today the publication, where formulas are printed in text editor, usually does not contain new results.

**The goals of development –** creation of methods and tools for optimization and simulation of a broad class of problems of optimum management of the development of production systems

### Components of development model using external resources

**1. Method of optimum aggregation.** Let us recollect the setting of tasks of nonlinear programming.

*Direct task* – maximization of total production at limitation of resources. The system of $N$ components, which uses some resource in the amount of $x_i$ and manufactures products in amounts of $x_i$ is considered: $y_i = fi(x_i)$; $i = 1, \ldots, N$, where $x_i$ – amount of resource, allocated for $i$-th element. It is needed to distribute the resource of $R$ so that to maximize total production:

$$F(x_1, x_2,.., x_N) = \sum_{i=1}^{N} fi(x_i) \text{ ; at limitation } Fs(x_1, x_2,.., x_N) = \sum_{i=1}^{N} fi(x_i) - Ys = 0 .$$

*Conjugate task*- minimization of total charges at limitation of total production level. We consider the same system of $N$ production elements. It is necessary to distribute loading of $Ys$ so that to minimize total charges:

$$Gs(x_1, x_2,.., x_N) = \sum_{i=1}^{N} x_i \text{ on condition } Fs(x_1, x_2,.., x_N) = \sum_{i=1}^{N} fi(x_i) - Ys = 0$$

*Management*- $x_i$ *or* $y_i = fi(x_i)$

*Methods of solution*. Essence of the known methods of nonlinear programming – finding function extremum of *N* variables at limitations, or "task of point selection in *N* – dimensional phase space" using terminology of Bellman [1]. The method of optimum aggregation replaces the task of finding the function extremum of many variables by the sequence of tasks of finding the function extremum of one variable [3 – 4].

*The first step* in the method of optimum aggregation is expansion of the task.

*Introduce the vector function of optimum distrbution of resources* $Dop(R)$, $0 \le R \le R\max$, where $R\max$ is maximum value of limitation. The components of this vector function set optimum by total production criterion distribution of resource.

*Introduce optimum production function* of the system $Yop(R) = \sum_{i=1}^{N} fi(Dop(R)_i)$.

Function $Yop(R)$ for each value of limitation on the resource *R* sets maximum efficiency of transformation of resource in a product.

*Formulate expanded optimization problems: N* of production functions are set, additive limitation on resource and additive criterion is total production; it is needed to find the optimum production function of the system $Yop(R)$ and vector function of the optimum distribution of resource $Dop(R)$.

*The second step is transition to the dimensionless variables of management.* Instead of variables of management $x_1, x_2, \ldots, x_N$ enter dimensionless variables $\alpha_1, \alpha_2, \ldots, \alpha_N$; where $\alpha_1 = x_1 \div R$; $\alpha_2 = x_2 \div R$ . Content of these variables – shares of resources for the development of relevant elements, the amount of shares equal one. Such formal replacement allows to prove that optimum production function (PF) will be the one, which bends the great number of certain production functions of system elements. The method of optimal aggregation substantiation is based on this fact. Enter the set of α- functions:

$$f\alpha(f1, f2, \alpha, x) := f1(\alpha \cdot x) + f2[(1-\alpha) \cdot x]$$

Optimum PF system of two elements will be the function, which bends a great number of $f\alpha(f1, f2, \alpha, x)$, $0 \le \alpha \le 1$, i.e. the result of application of $\max(\cdot)$ operation which is *associative* and *commutative*. For the system with the additive criterion optimum PF $FopN(f1, f2, \ldots, fN)$ has the property:

$$Fop3(f1, f2, f3) := Fop2(f1, Fop2(f2, f3)).$$

Programming facilities allow to realize *the binary operator of optimum aggregation* $f2o(f1, f2)$, which takes the pair of functions of *f1, f2* and returns optimum production function and corresponding vector function of optimum distribution of resource. The volume of calculations on this method *grows only linearly, but not exponentially. A method does not have limitations regarding the form of PF*, because maximum of function of one variable is found using direct exhaustive search method. The method, in essence, replaces the task of search of extremum with an algebraic task. *The operator of optimum aggregation generates algebra* with one associative and commutative operation, elements of which are arrays of variable dimension.

**2. Method of solution of optimum development variation task.** R. Bellman studied the structure of solutions for variation tasks of distribution and found a solution for the special case – Markovic problem. Essence of development problem is optimum distribution of current resources between accumulation and development – by creation of "production capacities". This base task was modified by such components:

- usage of maximum principle method , but not the method of dynamic programming;
- finding of maximum of Hamilton function by the method of direct exhaustive search;
- usage of the method of optimum aggregation for replacement of multidimensional object by

equivalent optimum one-dimensional object.

Let us briefly review the solution of one-dimensional problem of development by the principle of maximum method.

*Management object*: $\frac{d}{dt}x(t) = fin(y(t))$, where *x(t)* – rate of production , *y(t)* – rate of investments, $y(t) = x(t) \cdot u(t)$, $0 \le u(t) \le 1$ - normalized management, *fin(y(t))* function of investments reimbursement – non strictly monotonously increasing function.

*Boundary conditions* $x(0) = x0$ – starting rate of production, $T_p$ – planned period.

*Criterion* $J1 = \int_0^T x(t) \cdot (1 - u(t)) dt$ "accumulated income", *goal of optimization* $\min_{ut}(J)$ management which gives maximum of criterion, – income accumulated for planned period.

*Exact solution of the task*. We write down the task in a canonical way – add differential equation for criterion: $\frac{d}{dt}x(t) = fin(x(t) \cdot u(t))$; $\frac{d}{dt}J1(t) = x(t) \cdot (1 - u(t))$ .

Introduce denotations: $\frac{d}{dt}x(t) = fin(x(t) \cdot u(t)) = fx$ ; $\frac{d}{dt}J1(t) = x(t) \cdot (1 - u(t)) = fJ$ .

Write down the Hamilton function: $H(x,u) = \sum_{i=0}^{N} \psi_i f_i = \psi J \cdot fJ + \psi x \cdot fx$ .

Substitute right parts of differential equation and will get

$$H(x,u) = \psi J \cdot [x(t) \cdot (1 - u(t))] + \psi x \cdot fin(x(t) \cdot u(t)).$$

Write down equation for determination of functions.

$$\frac{d}{dt}\psi J(t) = -\frac{\partial}{\partial J}H(x,u) \ ; \ \frac{d}{dt}\psi x(t) = -\frac{\partial}{\partial x}H(x,u) \ .$$

Find the appropriate partial derivatives of $H(x, u)$

$$\frac{\partial}{\partial J}H(x,u) = 0 \ ; \ \frac{\partial}{\partial x}H(x,u) = \psi J \cdot (1 - u) + \psi x(t) \cdot u \cdot \frac{\partial}{\partial x} fin(u \cdot x).$$

On the basis of numerical solution of these equations Hamilton function is found

**3. Expansion of task of optimum development – credit strategies.** By unknown reasons Bellman did not examine variation task of distribution taking into account the use of external resources – credits. The use of external resources in the processes of development is a norm, therefore let us introduce another variable of management – rate of credits. We must determine now, besides *optimum strategy of development* (proportions of resources distribution between accumulation and development),still *optimum credit strategy* – volume of credits at each step of the process. We do not introduce one more variable – rate of credits return so far. We will consider the ordinary for banking sphere method of credits reimbursement: by equal shares from the moment, when a credit is taken, and to the end of period taking into account percents. We have two management variables: rate of credit *xkr(t)* and share of current resources *u1(t)*, directed in investments. Two functions of time *u1op(t), xkrop(t)*, which give maximum of accumulated income for planned period *Tp*. are to be found

The analysis of properties of Hamilton function allows to find the satisfactory approximations of this function in *space of strategies*, in fact we are interested only in positions of maximums of this function. Remember "physical sense" of Hamilton function – the "projection" of current managements on eventual result. Compare three expressions for Hamilton function : exact and approximated, without credits and with credits. Enter for reduction of expressions a variable "total current resources" $xs(t) = x(t) + xkr(t)$ and will write down alongside for comparison the expressions for Hamilton function: approximated, taking into account credits and exact one.

$$H(x,u) = x \cdot (1 - u) + fin(x \cdot u) \cdot (T - t) \ ;$$

$$H(xs,u) = xs \cdot (1-u) + fin(xs \cdot u) \cdot (T-t) - xkr \cdot [1 + prc \cdot (T-t)] \ ;$$

$$Ho(x,u) = x \cdot (1-u) + fin(x \cdot u) \cdot \psi n\left( x, u, \frac{\partial}{\partial y} fin(y) \right) \ ;$$

where $T$ – time of completion of process, $t$ – current time, $(T - t)$ – time to the end of the process, $x(t)$ – rate of production, $xkr(t)$ – rate of credits, $\psi n\left( x, u, \frac{\partial}{\partial y} fin(y) \right)$ – function of user, defined by means of numeral solution of equation for conjugate function.

**Analysis of optimum processes of development. Receipt of function of influencing of rate of credits.** Just as in physics, the powerful accelerator of elementary particles allows to find new results, so adequate reality and computational effective program of development processes modeling becomes the generator of new results. Let us consider the example of research of dependence of producer credit strategies on the rate of credits and efficiency of investments. For obtaining the influence functions 20 -100 runs simulation program were performed. Fig. 1 provides twq-level interface for analysis of influence functions . A feature of interface is that the user can choose a point on the function of influence and observe in details the process of development of the production system, which is represented in this point. A credits rate influence function is the "natural" function of demand on credits, conditioned by optimum credit strategy. This strategy is calculated on the base of mathematical model and actual economy mechanisms. Such model becomes *the generatorof new knowledge* about properties of the systems of this class. Fig. 2 shows the example of such situation: in the optimum process of development at reduction of credits rates , starting from some value of credits rate (in this case – 20%), the accumulated income of production grows, and the accumulated income of bank – falls. However; *the total profit of the system grows*, as interests of sides are not antagonistic: we have a game with a nonzero sum. The model of optimum development allows to calculate the size of indemnification to the bank of losses due to the decline of credits rate , in this case both sides win . Thus (Fig. 2) model suggests the way to the simultaneous gain increase of producer and bank due to the use of other mechanism of co-operation: *the profits of bank are proportional not only th volume of credits butvolumeof profits of production*. Separate analogue of the offered mechanism is the mechanism of enterprise corporization: profit on shares depends not only on the amount of shares but also on the income of enterprise.

**Optimization of strategies of credits reimbursement.** Examples of results presented in Fig 1;2 are valid for standard scheme of credits reimbursement (reimbursement of the body of credits and percents by equal parts before the end of the planned period). We take the following step different from the standard pattern of loans reimbursement by equal parts. *expanded problem of optimal developmentwasset and solved*: to control variables "share of total resource for development", "rate of credits" variable "rate of credits reimbursement" was added.

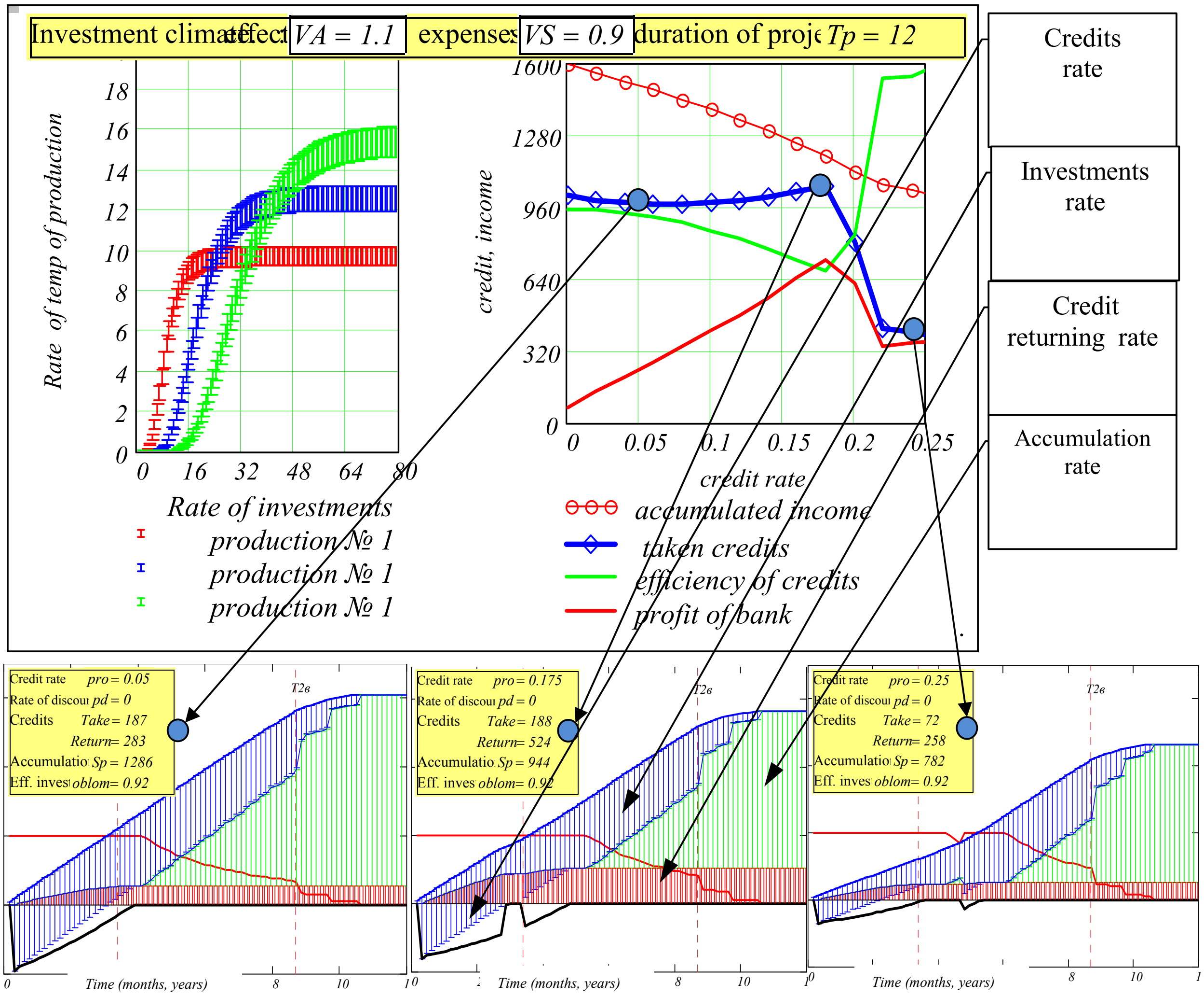


Fig. 1. Analysis of credits rate influencing function on the indexes of the project

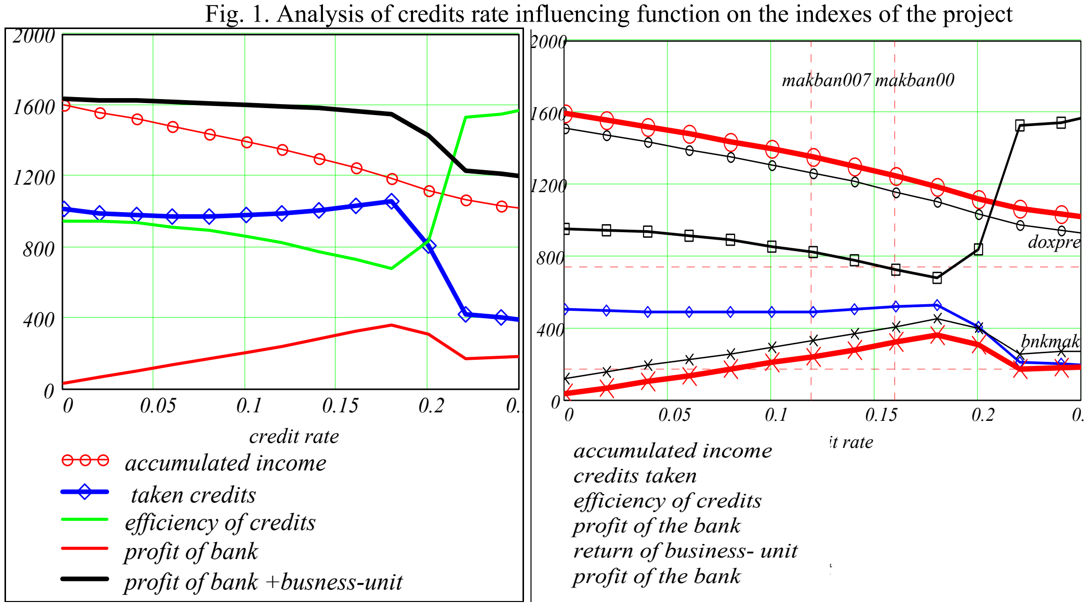


Fig. 2. Analysis of profit distribution in the system "producer-bank"

Fig. 3 presents the results of the simulation of optimum development strategy with two strategies of credits reimbursement: by equal shares and optimum strategy of credits reimbursement. The form of interface is convenient for analyst and specialist on management.

In Fig. 4 the alternative form of interface is presented – balanced, convenient and "natural" for a financier. These decisions can be replaced by approximate , convenient for practical realization. Verbal formula of optimum management development: – first all resources (own and borrowed) are directed into investments, optimum volume of investments is approximately constant, if the effects of mastering of production are not available in production system; – credits are taken in the volume necessary to supplement own resources to the optimum level of investments, crediting is halted when own resources are sufficient for development; – after ceasing crediting remaining part of investments is allocated for credits reimbursement; – the process of accumulation starts only after returning of debts.

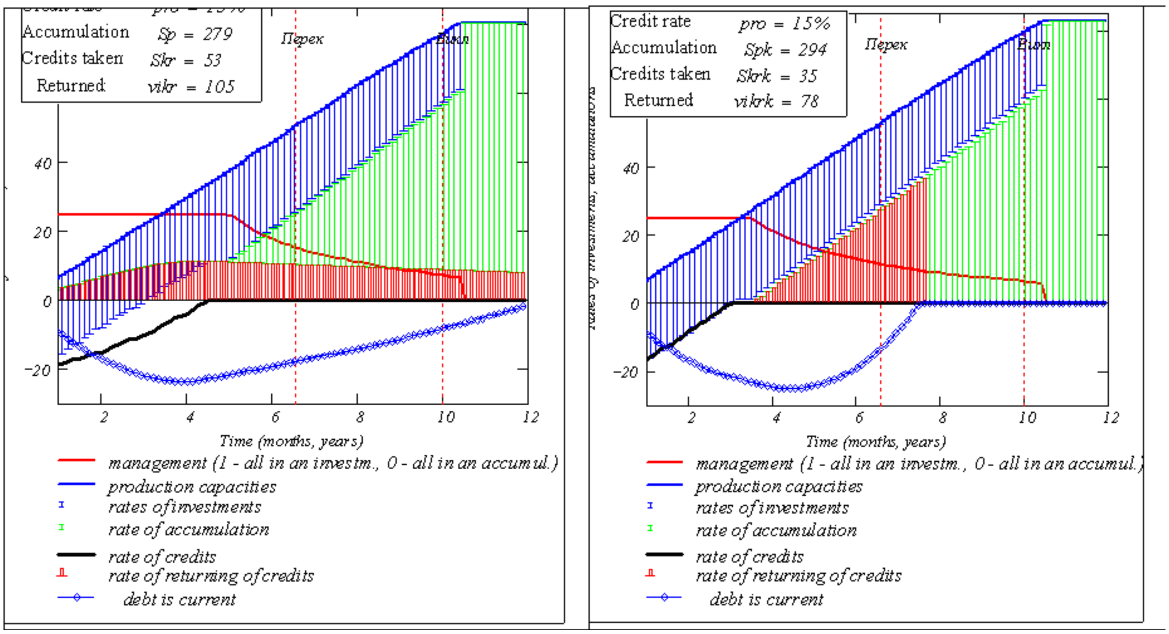

Fig. 3. Comparison of two strategies of credits reimbursement

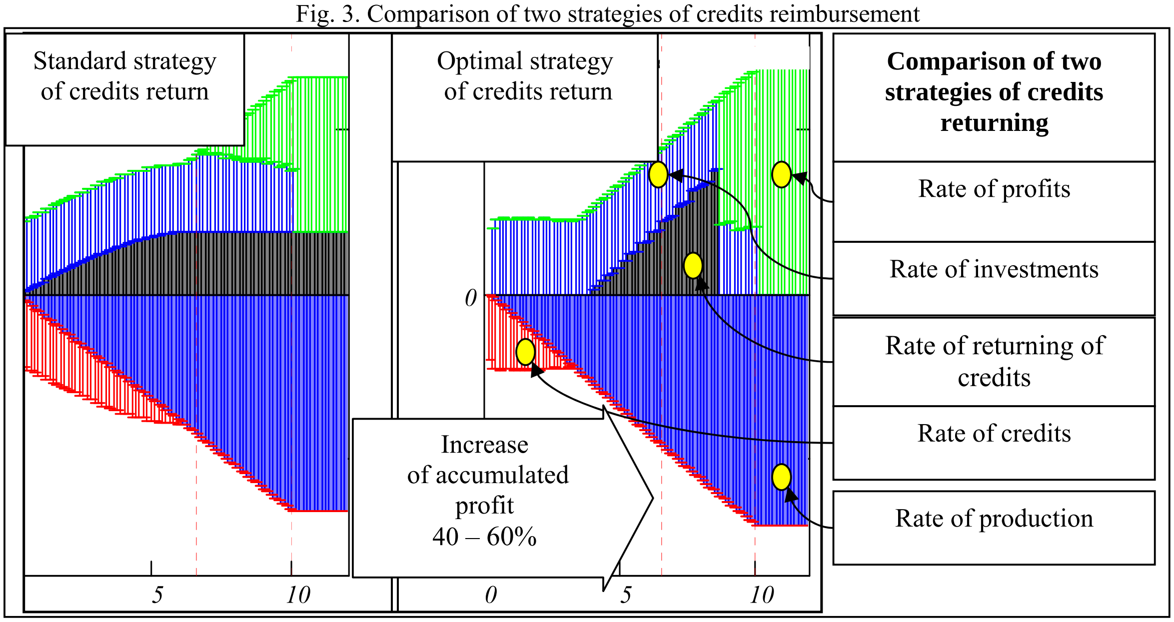

Fig. 4. Analysis of strategies of credits reimbursement. Balance form

**Results of development**. The exact and approximate solution of variation task of development taking into account the use of external resources is found, the model of process of development of the system of "N producers at the market of M products" is developed, the modules of construction of influence function and frequency distributions of risks are developed. Examples of results modeling, shown in Figs1-4 are indicators that the declared models have been developed, and program modules function . In the article final results of rather volume process of construction of mathematical models system of optimum development are considered . The complete theoretical substantiation of the approach to the solution of variation problems is performed in the environment of mathematical package using the tooling of symbolic computations and has a volume of 35 pages. This development is the module of the system of models of “N class of producers, M products, K consumers".

**Conclusions.** The basis of the obtained results is the effective solution of variation problem of development using the method of optimum aggregation and method of variation development problem solution taking into account the usage of external resources. The system of working codes and interfaces – "virtual reality” is developed. New results (properties of optimum processes of development) are obtained, these results are interesting for theory and useful for practical application :gap of optimum strategies of development; gap of credit strategies; reduction of credits demand at the low credits rates; – the terms of noncontradiction of interests of bank and producer are found; – solution on coordination of interests of sides is offered due to just distribution of profits.

Methodological results of research : – it is shown that credits not only increase the accumulated income of the project for planned period but also simplify process of development control, and the process itself is made a less risky; – the example of rational technology of constructing of new models for new tasks, oriented on technologies and resources of the Internet, in particular on "software as service" is suggested.

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