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INTERNATIONAL BY NURBS – CURVES IN MULTIDIMENSIONAL SPACE

The paper considers the construction of curves based on NURBS using basic Bernstain functions. The approach to construction of interpolation curves in multidimensional space, enabling to enlarge the spheres of their application is elaborated. The described technique of construction of multidimensional curves of such type allows to find efficient solution of the problem for spaces of larger dimensionality.

Key words*: interpolation Besier curves, basic functions, control points, NURBS, computer graphics, CAD.*

Nowadays rational Besier curves are used for curves construction in various CAD systems. It is very flexible tool, enabling to construct smooth splines of any order, form, as well as to perform local control over the curve. The curve is presented in parametric form, control points and weight coefficients of nodes are used for curve form control [1]. Rational non – uniform Besier curves are related to NURBS, basic Bernstain functions being their basis. NRBS – curves have wide practical application: they are used in computer graphics for drawing smooth curves exactly describing the shape of 2D objects in figures and drawings, for setting flat curve of rotating surfaces, as well as for modeling the trajectory of motion on surface and in space in the course of time.

Parametric presentation of the curve enables to use it for interpolation of data in multidimensional spaces. One of the main properties of the curve is that it passes only across the first and the last control points [1]. For modeling of curves on the plane the given fact is not a complex problem in case of visual construction of the curve. But for construction of NURBS curves in 3 D system of coordinates additional efforts aimed at visualization of control points selection on flat monitor are needed, whereas for coordinate system of higher order such scheme can not be used at all.

The goal of the given paper is enlargement the application sphere of approach aimed at construction of interpolation functions based on NURBS curves for multidimensional case.

NURBS are standard tools for geometry representation [2]. Main reason for wide application of NURBS are:

1. Common geometrical form both for standard geometrical forms and for object of arbitrary form.
2. Flexibility in construction of a larger number of various forms.
3. Possibility of rapid calculation by means of numerically stable and accurate algorithms.
4. Invariance for affine transformations

NURBS *p(t)* curve in general form is defined as:

$$p(t)=\frac{\sum_{i=0}^{n} p_i w_i N_{i,k}(t)}{\sum_{i=0}^{n} w_i N_{i,k}(t)}, \qquad (1)$$

where w – vector of weight coefficients; p – vector of control points of $n+1$ dimensionality; $N_{i,k}(t)$ – normalized basis functions of k power; t – parameter.

Among principle properties of the curve, constructed across the control points ($p_0, p_1,\ldots, p_n$), the following should be mentioned:

1. The curve interpolates extreme points. The curve starts at point p_0 and stops at points p_n. This property makes curve construction understandable.

2. Control points perform local control over the curve. The curve bends in zone of control point influence, depending on change of the point location, the remaining part of the curve is not

subjected to considerable changes.

3. The curve remains inside imaginary triangle, formed by control points[1].

Fig 1 shows the example of NURBS curve, built by four points.

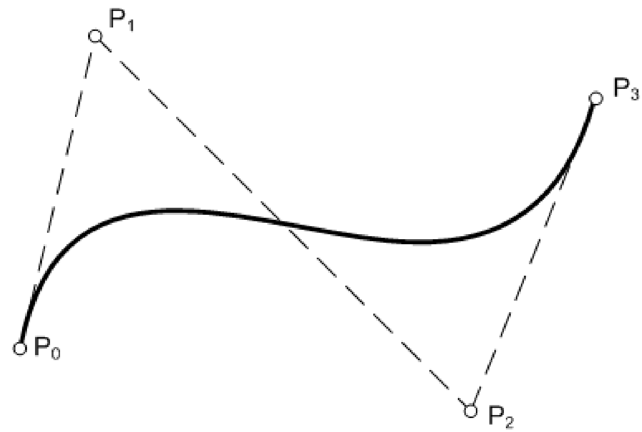

Fig 1 Example of NURBS curve, built by four points.

Basic functions set the degree of the curve and character of control point influence on the curve Weight coefficient make the curve rational, enabling more exactly to determine the degree of the given control point influence on the shape of the whole curve.

In the given paper subtype of NURBS, based on fundamental functions of Besier curves is used [1]. Fundamental Bernstain functions are used while formation of Besier curves:

$$B_i^n(u) = \binom{n}{i} u^i (1-u)^{n-i}, \tag{2}$$

where $\binom{n}{i} = C_n^i = \dfrac{n!}{i!(n-i)!}$ – binominal coefficient, also called formula for computation of combinations in combinatorics. Value of basis function (2) are calculated for $0 \le i \le n$, and $0 \le u \le 1$.

NURBS-curve on the basis of fundamental Bernstain functions is presented in formula(3):

$$p(t) = \frac{\sum_{i=0}^{n} \binom{n}{i} p_i t^i (1-t)^{n-i} w_i}{\sum_{i=0}^{n} \binom{n}{i} t^i (1-t)^{n-i} w_i}. \tag{3}$$

For problem solution, proceeding from the formula (3), the system of linear equations (4) is built, coordinates of control points in this system will be unknown, each space coordinate being considered separately. Having *n+1* node of interpolation, we will be able to compose and solve the system matrix of coefficients and matrix of absolute terms are specified by the formula (4).

$$A_{i,j} = \binom{n}{j} t_i^j (1-t_i)^{n-j} w_j, \; B_i^k = y_i^k \sum_{j=0}^{n} \binom{n}{j} t_i^j (1-t_i)^{n-j} w_j, \tag{4}$$

where y^k – vector of interpolation nodes for k-th space coordinate; A – matrix of coefficients; B^k – vector of absolute terms for k- th space coordinate .

Since the first and the last interpolation nodes are identical to key point NURBS, then the number of equations system (4) can be reduced by 2, replacing the matrix of absolute terms of matrix by the matrix, as it is shown in formula(5).

$$B_i^k = y_i^k \sum_{j=0}^{n} \binom{n}{j} t_i^j (1-t_i)^{n-j} w_j - y_0 (1-t_i)^n w_0 - y_n t_i^n w_{n.} \qquad (5)$$

The solution of the system will be the set of control points, by which NURBS curve can be constructed, which will solve interpolation problem in the space of any dimensionality.

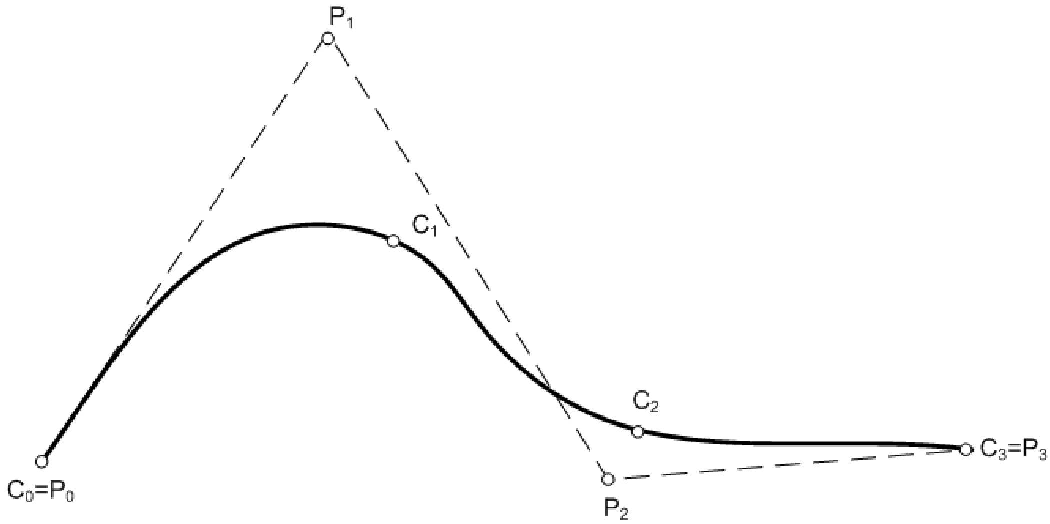

Fig 2. Solution of interpolation problem by NURBS curves.

Fig 2 shows the example of the given problem solution, C_0, C_1, C_2, C_3, – задані preset interpolation nodes, P_0, P_1, P_2, P_3 – key points to be determined, зfound as a result of solution of the system, interpolation NURBS – curve is constructed by these points.

Matrix*A* does not depend on the values of interpolation nodes, this allows to solve the problem for multidimensional space applying Gause – Gordon method.

Gausse – Gordan method is modification of Gausse method for solution of linear equatations system. This method allows to reduce coefficient matrix to diagonal form. In this case, not only lines located below, but also lines, located above the diagonal are eliminated. The given method facilitates the obtaining of solution, but is accompanied by increase of computation volume [3].

Forward action of Gausse – Gordon method requires more powerful computational resources but reverse motion, in which the vector of system solution is calculated, becomes simpler. Proceeding from the properties of the method, we can make conclusion, that the method allows to solve the class of the problems , where it is necessary to obtain the solution of the systems of linear equations with identical matrices of coefficients and different vectors of absolute terms .This technique allows during one passage to solve the problem of control points search for all dimensionalities of multidimensional space, as matrices of absolute terms for these points will be identical Elimination of the coefficients in forward motion of the method must be performed over the matrix (6).

$$G = [A\,B^0\,B^1 \ldots B^{k-1}] = \begin{bmatrix} a_{00} & a_{01} & \ldots & a_{0n} & b_0^0 & b_0^1 & \ldots & b_0^{k-1} \\ a_{10} & a_{11} & \ldots & a_{1n} & b_1^0 & b_1^1 & \ldots & b_1^{k-1} \\ & & . & . & . & & & \\ a_{n0} & a_{n1} & \ldots & a_{nn} & b_n^0 & b_n^1 & \ldots & b_n^{k-1} \end{bmatrix}, \qquad (6)$$

where k – dimensionality of the space, the curve is calculated for,

B^i – vector – column of absolute terms of the system for i-th dimensionality of the space.

Computational complexity of forward motion Gausse algorithm with the selection of leading line is evaluated by O(n) 4. Proceeding from the form of matrix G(formula 5) the expansion of the problem by means of space increase by one order(addition of one column in the matrix G) will not result in considerable complication of the problem.

Conclusions

The approach to curves construction, suggested in the given paper, enables to apply subtype of NURBS – curves, constructed on basis of Bernstain functions, for the solution of interpolation problem, enlarging the sphere of curves application for problems solutions in multidimensional spaces. The solution obtained can be applied while construction of objects routes in space, taking into account time as parameter, as well as for solution of the other complex parametric interpolation problems. In this case control points are set directly on interpolation curve that considerably simplifies with work with the given type of curves at their space modeling. Application of the described method for construction curves provides the possibility of rapid and efficient solution of the problem without substantial increase of calculation complexity if problem dimensionality grows.

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