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MATHEMATICAL MODEL FOR ANALYSIS OF TRANSIENT STATES OF ASYNCHRONOUS MOTORS

The mathematical model of asynchronous motor, in which saturation and iron loss are considered, stator and rotor currents are used as a state of variables is developed.

Keywords: *asynchronous motor, mathematical model, transitive states.*

Case study and statement of the problem

Choice of mathematical models of electric motors is important and often determining for problems solution of analysis of transient states of blocks of electrical networks containing powerful asynchronous or synchronous electric drives. The most effective at solution of such problems is use of mathematical models in orthogonal co-ordinates where orthogonal voltages and currents of electric machines are used as output variables. Such co-ordinate system provides simple analytical connection between variables parameters of electric machines and elements of electrical network by means of differential equations of the first order.

For mathematical models of electric motors during analysis of transient states of electrical networks blocks the following simplifications usually are accepted:

- magnetic core and iron loss saturation are neglected;
- magnetic field is conventionally divided into two parts - base field and field of dissipation (dispersion);
- base field is considered to be plane-parallel;
- gear zones are presented by continuous non-isotropic magnetic layers ;
- only the first space harmonics of magnetomotive forces are taken into account;
- windings of phases are considered as ones , that are linked only with the flux of the first induction harmonic of air-gap.

In this article the mathematical model of asynchronous motor (AM) is offered, in which saturation of magnetic core and loss in it are considered, however, unlike [1], in the capacity of state variables, instead of the basic magnetic flux linkage, rotor current is used. Using of stator and rotor currents will allow to determine control parameters during transients of frequency driven AM with short-circuited rotor, as well as AM with phase-wound rotor more precisely.

Substantiation of results

Asynchronous machine differential equations in generalized orthogonal coordinates with random frequency of its rotation are obtained on the basis of law of voltage and e.m.f. balance

$$\begin{aligned} \frac{d\Psi_s}{dt} &= \mathbf{U}_s + \mathbf{\Omega_s \Psi_s} - \mathbf{R_s I}_s; \\ \frac{d\Psi_r}{dt} &= \mathbf{U_r} + \mathbf{\Omega_r \Psi_r} - \mathbf{R_r I_r}, \end{aligned} \tag{1}$$

where $\mathbf{\Psi_s}, \mathbf{U}_s, \mathbf{I}_s$; $\mathbf{\Psi_r}, \mathbf{U_r}, \mathbf{I_r}$ - vectors of complete magnetic-fluix linkages, voltage and currents of stator and rotor correspondingly ; $\mathbf{\Omega_s}, \mathbf{\Omega_r}$ - matrixes of rotation frequencies; $\mathbf{R}_s, \mathbf{R}_r$ - matrixes of resistances of stator and rotor correspondingly .

In complete form the expressions (1) can be presented as [1]:

$$\begin{bmatrix} \frac{d\Psi_{s\alpha}}{dt} \\ \frac{d\Psi_{s\beta}}{dt} \\ \frac{d\Psi_{r\alpha}}{dt} \\ \frac{d\Psi_{r\beta}}{dt} \end{bmatrix} = \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \\ u_{r\alpha} \\ u_{r\beta} \end{bmatrix} + \begin{bmatrix} 0 & \omega_k & 0 & 0 \\ -\omega_k & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_k - \omega_r \\ 0 & 0 & -(\omega_k - \omega_r) & 0 \end{bmatrix} \begin{bmatrix} \Psi_{s\alpha} \\ \Psi_{s\beta} \\ \Psi_{r\alpha} \\ \Psi_{r\beta} \end{bmatrix} - , \qquad (2)$$

$$- \begin{bmatrix} R_s + R_m & 0 & R_m & 0 \\ 0 & R_s + R_m & 0 & R_m \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix},$$

Vectors of complete magnetic-flux linkages of stator and rotor we will express through the vector of basic magnetic-flux linkage and vectors of magnetic-flux linkages of dispersion of stator and rotor correspondingly

$$\mathbf{\Psi}_s = \mathbf{\Psi}_\delta + \mathbf{L}_{\sigma s}\mathbf{I}_s; \;\; \mathbf{\Psi}_r = \mathbf{\Psi}_\delta + \mathbf{L}_{\sigma r}\mathbf{I}_r, \qquad (3)$$

Where $\mathbf{\Psi}_\delta$ - vector of the basic magnetic-flux linkage; $\mathbf{L}_{\sigma s}$, $\mathbf{L}_{\sigma r}$ - matrixes of inductances of stator and rotor dispersing correspondingly.

The vector of the basic magnetic- flux linkage can be expressed in terms of magnetization current vector and vectors of stator and rotor currents

$$\mathbf{\Psi}_\delta = \mathbf{I}_m \mathrm{L} = (\mathbf{I}_s + \mathbf{I}_r)\mathrm{L}, \qquad (4)$$

where $\mathbf{I}_m$ - vector of magnetization current; $\mathrm{L} = \mathrm{L}_0$ - static inductance of magnetization,

$$L = \frac{\psi_m}{i_m} = L(\psi_m).$$

Let us differentiate the basic magnetic-flux linkage in time

$$\frac{d\Psi_\delta}{dt} = L\frac{d\mathbf{I}_m}{dt} + \mathbf{I}_m\frac{dL}{dt}. \qquad (5)$$

The derivative $\frac{dL}{dt}$ we will note as complete

$$\frac{dL}{dt} = \frac{dL}{di_m}\left(\frac{\partial \mathrm{i}}{\partial \mathrm{i}_\alpha}\frac{\partial \mathrm{i}_\alpha}{\partial t} + \frac{\partial \mathrm{i}}{\partial \mathrm{i}_\beta}\frac{\partial \mathrm{i}_\beta}{\partial t}\right). \qquad (6)$$

Differentiating static inductance on the current, we have

$$\frac{dL}{di_m} = \frac{L_\partial - L_0}{i_m}, \qquad (7)$$

where L_∂ - incremental inductance which is determined by curve of magnetization $L_\partial = \frac{d\psi_m}{di_m} = L_\partial(\psi_m)$.

Taking into account that components of basic magnetic linkage

$$\Psi_{\delta\alpha} = \mathrm{L}\, I_{m\alpha}; \;\; \Psi_{\delta\beta} = \mathrm{L}\, I_{m\beta}, \qquad (8)$$

and the module of basic magnetic linkage

$$\Psi_{\delta m} = \sqrt{\Psi_{\delta\alpha}{}^2 + \Psi_{\delta\beta}{}^2}\ , \tag{9}$$

we will obtain the detailed expression (5) in the form

$$\begin{bmatrix} \dfrac{d\Psi_{\delta\alpha}}{dt} \\ \dfrac{d\Psi_{\delta\beta}}{dt} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} \dfrac{di_{s\alpha}}{dt} + \dfrac{di_{r\alpha}}{dt} \\ \dfrac{di_{s\beta}}{dt} + \dfrac{di_{r\beta}}{dt} \end{bmatrix} \quad , \tag{10}$$

where L_{11}, L_{12}, L_{22} - inductances of magnetization circuit :

$$L_{11} = L_0 + (\ L_\partial - L_0\ )\left(\frac{i_\alpha}{i_m}\right)^2;\ L_{12} = (\ L_\partial - L_0\ )\left(\frac{i_\alpha i_\beta}{i_m{}^2}\right);\ L_{22} = L_0 + (\ L_\partial - L_0\ )\left(\frac{i_\beta}{i_m}\right)^2.$$

Substituting (10) in (2) and taking into consideration (4), we will obtain the following system of differential equations

$$\begin{bmatrix} L_{\sigma s} + L_{11} & L_{12} & L_{11} & L_{12} \\ L_{12} & L_{\sigma s} + L_{22} & L_{12} & L_{22} \\ L_{11} & L_{12} & L_{\sigma r} + L_{11} & L_{12} \\ L_{12} & L_{22} & L_{12} & L_{\sigma r} + L_{22} \end{bmatrix} \begin{bmatrix} \dfrac{di_{s\alpha}}{dt} \\ \dfrac{di_{s\beta}}{dt} \\ \dfrac{di_{r\alpha}}{dt} \\ \dfrac{di_{r\beta}}{dt} \end{bmatrix} = \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \\ u_{r\alpha} \\ u_{r\beta} \end{bmatrix} -$$

$$- \begin{bmatrix} R_s + R_m & -\omega_k (L_{\sigma s} + L_0) & R_m & -\omega_k L_0 \\ \omega_k (L_{\sigma s} + L_0) & R_s + R_m & \omega_k L_0 & R_m \\ 0 & -(\ \omega_k - \omega_r\ ) L_0 & R_r & -(\ \omega_k - \omega_r\ )(L_{\sigma r} + L_0) \\ (\ \omega_k - \omega_r\ ) L_0 & 0 & (\ \omega_k - \omega_r\ )(L_{\sigma r} + L_0) & R_r \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix}. \tag{11}$$

T-shaped of AM equivalent circuit is adequate to considered mathematical model.

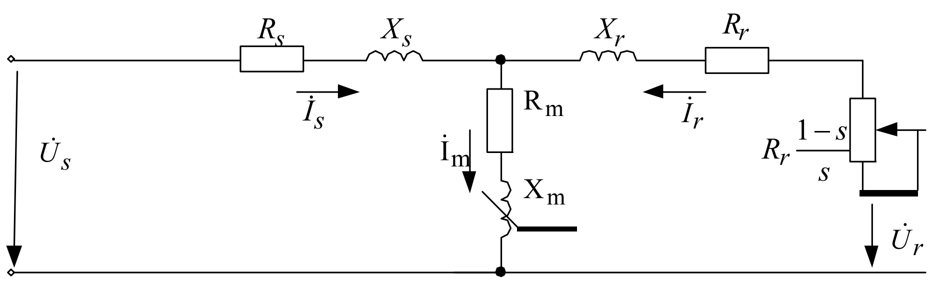

Fig. 1. Asynchronous motor equivalent circuit

For analysis the differential equations (11) must be supplemented with the equation of mechanical motion

$$\frac{d\omega_r}{dt} = \frac{\frac{3}{2} p_0 L_0 (\ i_{s\beta} i_{r\alpha} - i_{s\alpha} i_{r\beta}\ ) - M(\ t\ )}{J}, \tag{12}$$

where $M\ (t)$ - mechanical moment; J - moment of inertia; p_0 – number of machine poles pairs .

Differential equations (11), (12) - model of saturated asynchronous machine in orthogonal co-ordinates which can be used for analysis of transient states. Advantage of the model is that the use of stator and rotor currents will allow to determine the parameters of control during transients of frequency driven AM with short-circuited rotor, as well as AM with phase-wound rotor more precisely.

Conclusions

The mathematical model of asynchronous motor is elaborated in which saturation of magnetic circuit and loss in it is considered, and in the capacity of state variables stator and rotor currents are used .

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