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A. Morgun, Dr. Sc. (Eng.), Prof.; A. Nitsevich; I. Met

FORMATION OF INFLUENCE MATRIX OF BOUNDARY ELEMENTS METHOD TAKING INTO ACCOUNT ACTION OF VERTICAL AND HORIZONTAL STRESSES

The paper researches the influence of appearance of horizontal movements of the influence of vertical and horizontal loading on the piles and the expediency of the accounting of the numerical computation of radial tension on the value of the pile load-carrying ability.

Keywords: *mode of deformation, soils, numerical simulation, boundary elements method.*

Introduction

Acute social necessity leads to the increase of volumes and scales of housing construction and construction engineering. Grouth of volumes of construction becomes possible only in conditions of rational usage of financial, material resources, labour force. As we know: the estimated cost of foundations is about 40% of the total cost of the project. Due to specific geological conditions of Ukraine it is expedient to use piles foundations in industrial and housing construction. But in spite of wide application of piles foundations, especially for construction of high – rise structures, specific characteristics of their interaction with basement as well as theoretical methods of piles computation have not been sufficiently studied from the point of view of efficient economic and reliable design solutions. As a matter of fact this can be explained by the complexity of processes occurring in the soil when the pile penetrates in it and certain peculiarities of basement and pile, interaction.

Since these phenomena have not been studied sufficiently in design practice rather simple models are used. These models cannot completely reflect real processes of pile – basement interaction. Thus, the suggested solutions cannot be regarded as perfect ones.

Analysis of recent achievements

Intensive development and large – scale application of computers considerably narrowed the gap between mathematical problems and applied research; made their mutual influence more efficient. The advent of new powerful method of investigation – numerical experiment – as never before closely connected physical content of the problem, mathematical formulation and numerical method of solution. Problem dealing with forecast of piles foundation behavior can be solved, applying modern numerical methods and computers.

Among modern numerical methods the main group intended for solution of problems dealing with construction mechanics and mechanics solid deformed body is the method of finite elements (MFE) and method of boundary elements (MBE). Nowadays method of finite elements is one of the most widely used numerical methods aimed at solution of problems of media mechanics. The given method attracted attention of the researchers first of all due to the fact that the solid medium has a property to be divided into a number of elements, which can be considered as its separate parts. The above-mentioned method realizes the idea suggested by Poisson – solution of the problem regarding stressed-strained state of complex construction by considering its elementary fragments.

Method of boundary elements is an alternative approach that uses surface digitization of examined object, therefore this method is the most efficient for three-dimensional problems of foundation engineering. Solution of the given problem using MBE method is possible if corresponding fundamental solution is available (exact or approximate Green function). The problems is reduced to transformation of initial calculated differential equations to integral equation the solution of which is much more easier. Such operation provides the possibility to obtain the system of algebraic equations, which belongs to boundary area.

Problem statement, definition relations

The paper considers using numerical method of boundary elements axially symmetric problem aimed at definition of stressed-strained state and carrying capacity of the pile being deepened in the soil on the action of vertical loading. In general case stressed state emerges along lateral surface of prismatic or pyramidal pile and under its edge (Fig 1) tangential stresses τ_s along lateral surface of the pile,

τ_r - radial stresses along lateral surface of the pile,

σ_l - normal stresses under pile edge.

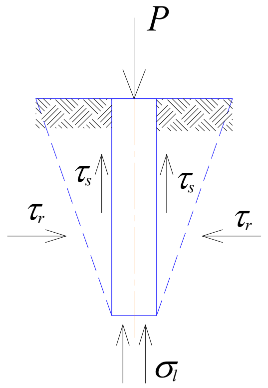

Fig 1. Stressed state of the pile from the vertical loadings

Under the action of the values of there stresses the displacements of lateral surface and edge points determined by fundamental Mindline solution for half-space will appear [1].

The values of these displacements (from single values of stresses) compose the matrix of influence of MBE Kij which from the point of view of engineering mechanics is a classical matrix. Integral boundary equation of pile equilibrium in the soil, obtained by K.Brebbija [2] has the following from an:

$$C_{ij}(\xi)\cdot U_j(\xi)+\int_{\Gamma}\rho^*_{ij}(\xi,x)\cdot U_j(x)\partial\Gamma(x)=\int_{\Gamma}U^*_{ij}(\xi,x)\cdot\rho_j(x)\partial\Gamma(x)+\int_{\Gamma}U^*_{ij}(\xi,x)\cdot b_j(x)\partial\Omega(x), \quad (1)$$

where $U^*_{ij}(\xi,x)$, $\rho^*_{ij}(\xi,x)$ - displacements and stresses emerging in point x in j^{th} direction and applied in point ξ (fundamental Mindline solutions for half- space); $U_j(x)$, $\rho_j(x)$ - displacement and stresses, emerging on body boundary (i.e., on lateral surface and on the surface of pile bottom edge).

Form of the matrix (1)

$$[K]_{ij}\cdot\{\Phi\}_j=\{W\}_i, \quad (2)$$

where $[K]_{ij}$ - global matrix of influence coefficients taking into account the interaction of pile with the base.

In this class of problems matrix [K] is always non singular with prevailing diagonal coefficients; $\{W\}_i$ – vector column of absolute terms of the system of Linear algebraic equations; from the point of view of physics – these are preset displacement of the points of pile lateral surface and points of the plane of pile bottom edge;

$\{\Phi\}_j$ – vector-column of the unknowns, which determine stress tangent lines $(\tau_s)_i$ (i=1,…n1). During computation of pile resistance, radial stresses $(\tau_\tau)_i$ (i=1,…n1) and normal stresses (σ_l) (i=2n1+1…n).

In stretched - out view (2) it can be presented in the following from

$$\begin{Bmatrix} W_S \\ U_S \\ W_b \end{Bmatrix}=\begin{Bmatrix} K_{SS} & K_{RS} & K_{BS} \\ K_{SU} & K_{RU} & K_{BU} \\ K_{SB} & K_{RB} & K_{BB} \end{Bmatrix}\cdot\begin{Bmatrix} \tau_S \\ \tau_R \\ \sigma_l \end{Bmatrix} \quad (3)$$

where K_{SS} – matrix of vertical displacements of lateral surface points from tangential stresses along lateral surface τ_s . The point of application of single load ξ is vertically on the lateral surface, control point X is also on the lateral surface;

K_{BS} – matrix of vertical displacements of the points of pile lateral surface from edge σ_l normal stresses. Point ξ - on vertical edge, point X – on lateral surface;

K_{SB} – matrix of vertical displacements of edge points from τ_s. Point ξ - on vertical lateral surface, X – on the edge.

K_{BB} – matrix of vertical displacements of edge points from σ_l. Point ξ - on the edge by verticality, point – on the edge.

K_{RS} – matrix, that includes coefficients on vertical displacements of pile lateral surface nodes in case of loading of lateral surface elements arch radial stresses τ_r. Point ξ – on lateral surface.

K_{SU} – matrix, the coefficient of which represent the connection between horizontal displacements of pile lateral surface nodes, when lateral surface I s loaded by vertical stresses. Point ξ - on lateral surface by verticality, point X – on lateral surface.

K_{RU} – matrix, that includes influence coefficients which represent dependence between horizontal displacements of pile lateral surface nodes when elements of lateral surface are loaded with horizontal stresses τ_r. Point ξ - on lateral surface by horizontally, point X – on lateral surface;

K_{BU} – matrix, coefficient of which represent dependence of horizontal displacement of bottom edge are loaded which vertical stresses σ_l. Point ξ - on the edge by verticality, point X – on lateral surface;

K_{RB} – matrix, coefficient of which represent connection between vertical displacements of pile bottom edge nodes, when elements of lateral surface are loaded with radial stresses τ_r. Point ξ – on lateral surface by horizontally, point X – on the edge.

In equations (2)-(3), which at the stage of digitization are written for each boundary node of edge and pile lateral surface, stresses on pile surface (τ_s, τ_r, σ_l) are unknown values, boundary conditions are defined by displacements of nodes on pile surface (W_s, U_s, W_b)).

The most labour – consuming part of the solution is computation of sub matrices the coefficients $(K)_{ij}$. Coefficients of these sub matrices are calculated by means of numerical integration of Mindline solution using Gausse quadrature formulas.

In order to take into account in numerical computation applying the method of boundary elements, the influence on pile carrying capacity the presence of three types of stresses along lateral surface and edge *Ts, Tr,* V_l the arrangement of influence matrix of MBE for pyramidal pile having the length 2.7m (60*60) (7*7) was carried out, the influence of its components on final result was analyzed. Matrix of influence was arranged in accordance with Mindline analytical solutions aimed at defining stresses – strained state in half – space under the action of a single source (P=1) both vertically and horizontally. The calculation takes into account the fact that the pile is absolutely rigid and displacements of pyramidal pile nodes located on lateral surface and bottom edge of the pile equal to displacement of upper edge of the pile. In calculation of pyramidal pile resistance value We was assumed to be equal the value of subsidence, when there exists linear dependence between pile resistance and its subsidence. As the analysis of experimental research of single pyramidal piles shows, value We within the limits of linear dependence in first approximation may be chosen to be equal 1–1.5cm

For the solution of the problem, lateral surface and edge of pyramidal pile were digitized by ten boundary elements. Linear approximation of stresses by the length of each boundary element was performed.

For each boundary node equation (2) was written, that led to creation of the system of linear algebraic equations of fifteenth order. For numerical realization of the problem on algorithmic language Pascal was composed. In this program SLAP radicals were determined applying Gausse method of the uniform division.

As a result of solution of the system of algebraic equations values of tangential stressed τ_s, radial stresses τ_r on lateral surface of pyramidal pile and normal stresses on the plane of lower ring σ_l. was obtained. After determination of stresses the force of pile resistance under the bottom edge (Pedge), force of pile resistance on lateral surface of pile (Plat) and total carrying capacity of pile was obtained an:

$$P_z = \int_L 2\pi\alpha\tau_s dh_l + \int_0^{a_n} 2\pi \in \sigma_l dh_2 \,, \tag{4}$$

where a_n - radius of pyramidal pile within lateral surface; h_1 - height of boundary element on lateral surface; ϵ – radius of the edge; h_2 - height of boundary element on pile height.

Coefficient of influence matrix K are given in Fig 2a, b, c. They are the line of influence of:
vertical displacements of lateral surface (Fig 2a)
horizontal displacements of lateral surface (Fig 2b)
vertical displacements of the edge (Fig 2c) from the action of single values τ_s, τ_r, σ_l.

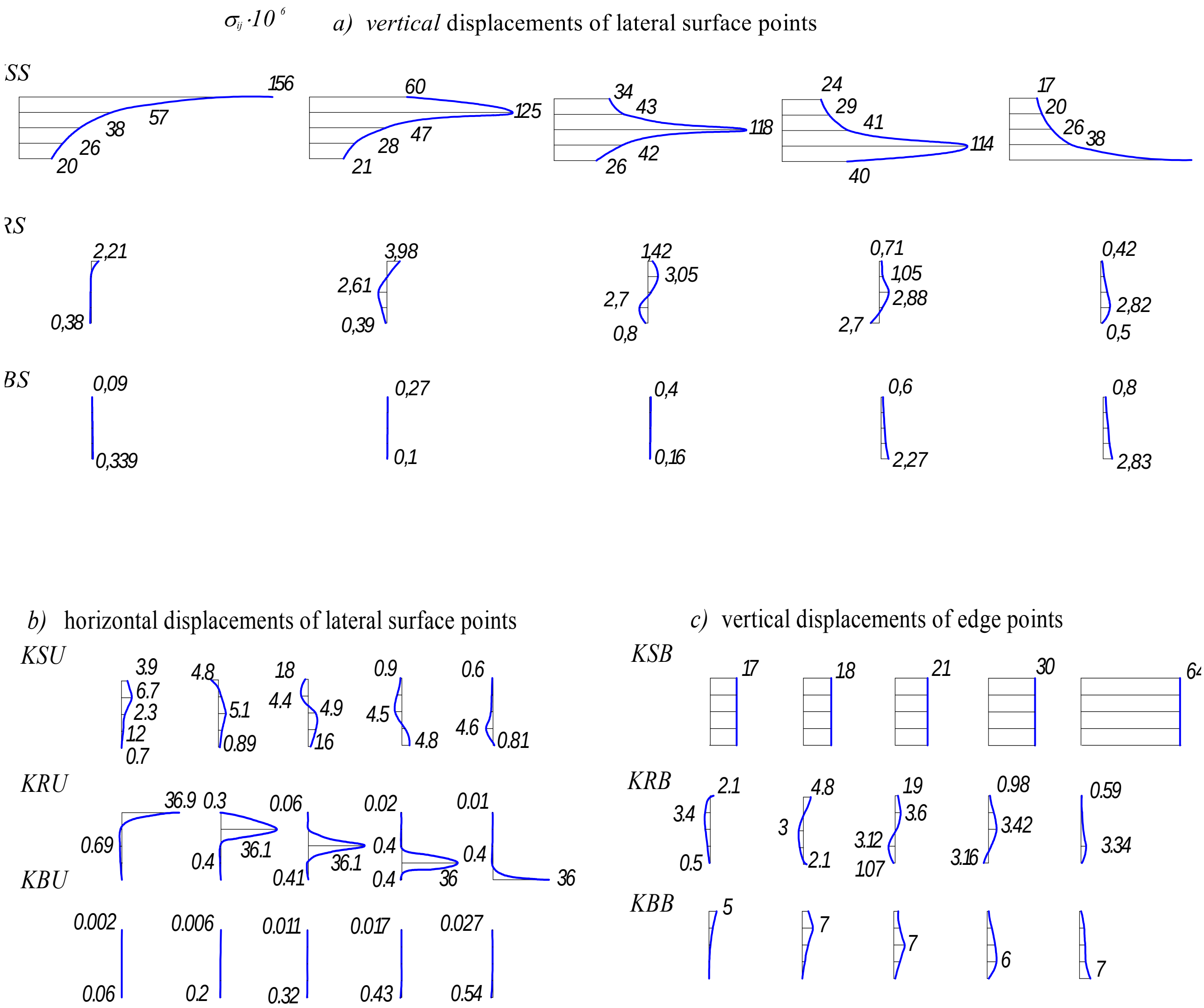

Fig 2. Coefficients of MBE influence matrix

Proceeding from numerical analysis of K_{ij} coefficients we should note (Fig 2) that the greatest influence on carrying capacity of the pile is exercised by vertical displacements due to the action of tangential stresses τ_s. Ordinates of these lines of influence are two order than ordinates of displacements due to the action of normal stresses V pile edge. Ordinates of horizontal displacements due radial stresses τ_r are considerable.

Numerical analysis of coefficients shows that lines of influence of displacements from horizontal forces τ_r have sections with positive and negative values of ordinates and when they are taken into consideration, give smaller values of matrix K_{ij} coefficients. When carrying capacity of the pile is defined, such matrix K_{ij} gives expected value of pile carrying capacity – 282 kN. Experimental data [3] – 280 kN. Numerical experiment was carried out in case of neglecting in matrix K_{ij} the action of radial stresses τ_r. In this case matrix K_{ij} is composed of four submatrices:

$$\begin{vmatrix} K_{SS} & K_{BS} \\ K_{SB} & K_{BB} \end{vmatrix}. \tag{5}$$

Carrying capacity of pile in this case was 272 kN. Since dependence between carrying capacity of pile that is integrated expression of stresses [4] and matrix K_{ij} coefficients is inverse (1), then greater carrying capacity corresponds to smaller values of K_{ij} matrix.

Conclusions

1. In numerical calculations of piles stressed – strained state on the action of vertical loadings, taking into account the influence of all nine submatrices in influence matrix gives the result closer to the experiment, approximates the result of modeling to real situation.

2. Neglecting the influence of horizontal stresses τ_r in case of vertical loading of pile (matrix K_{ij} consists of four submatrices) forecasts the decreased carrying capacity, that that is safety factor is within 3 – 4%.

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Alla Morgun – Head of the Department;

Andrey Nitsevich **–** Master student of the Department;

Ivan Met **–** Master student of the Department.

The Department for Industrial and Civil Engineering, Vinnytsia National Technical University.